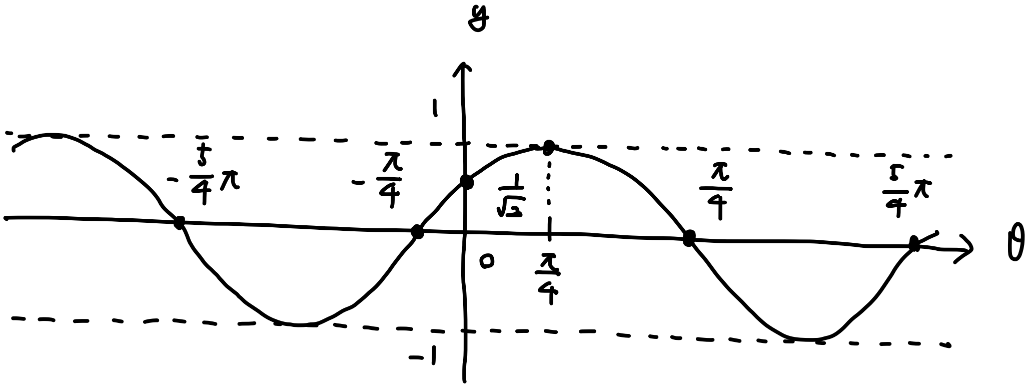
第6回答え

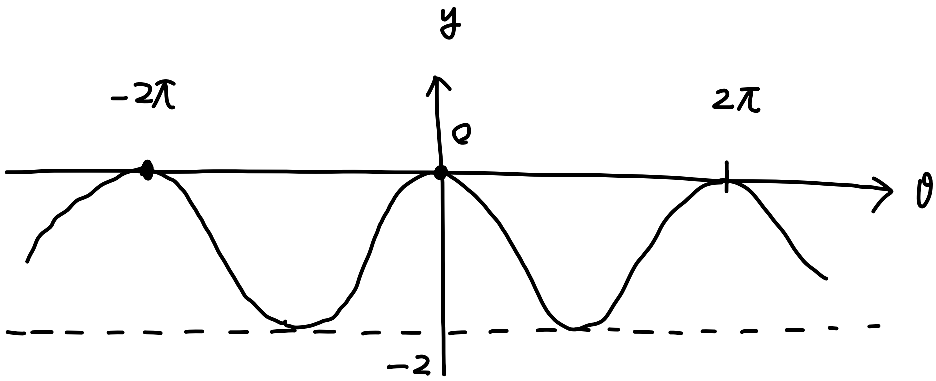
1. A : 1, B : $\frac{\pi}{2}$, C : $-\frac{1}{2}$, D : $\frac{5}{2}\pi$, E : $\frac{\pi}{6}$, F : $\frac{5}{6}\pi$, G : 2π

2. (1)



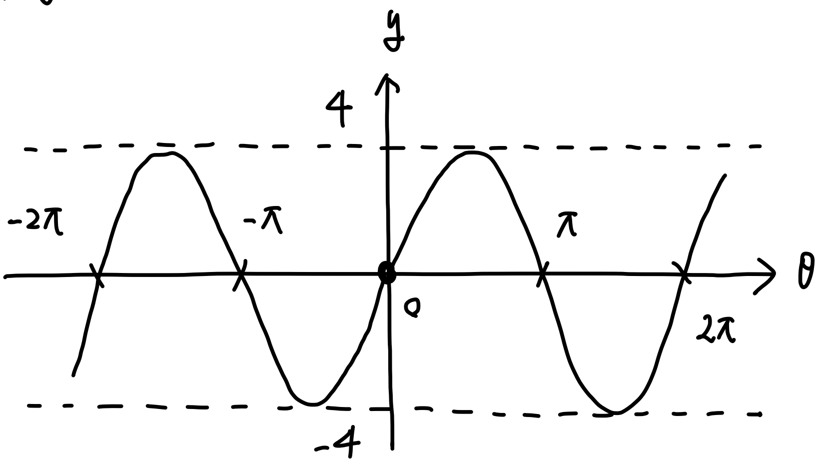
周期 : 2π, $y=\cos\theta$ を θ 軸方向に $+\frac{\pi}{4}$ 平行移動したグラフ。

(2)



周期 : 2π, $y=\cos\theta$ を y 軸方向に -1 平行移動したグラフ。

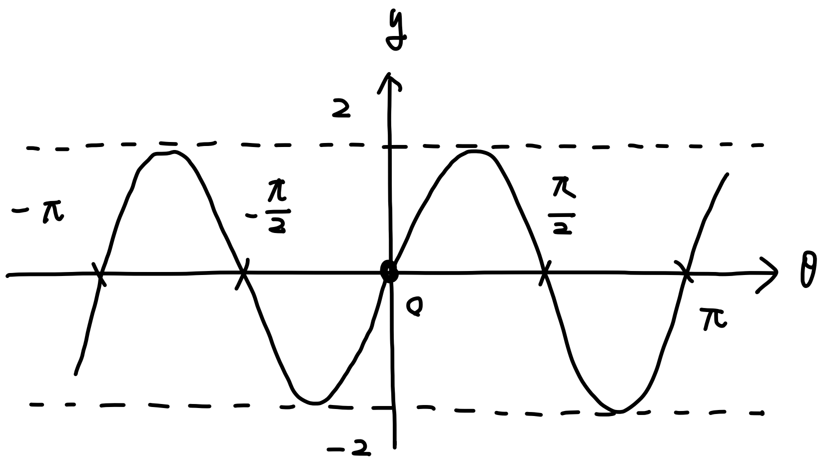
2. (3) $y = 4\sin\theta$



周期：2π，$y=\sin\theta$ を y 軸方向に 4 倍した

グラフ。

(4) $y = 2\sin 2\theta$



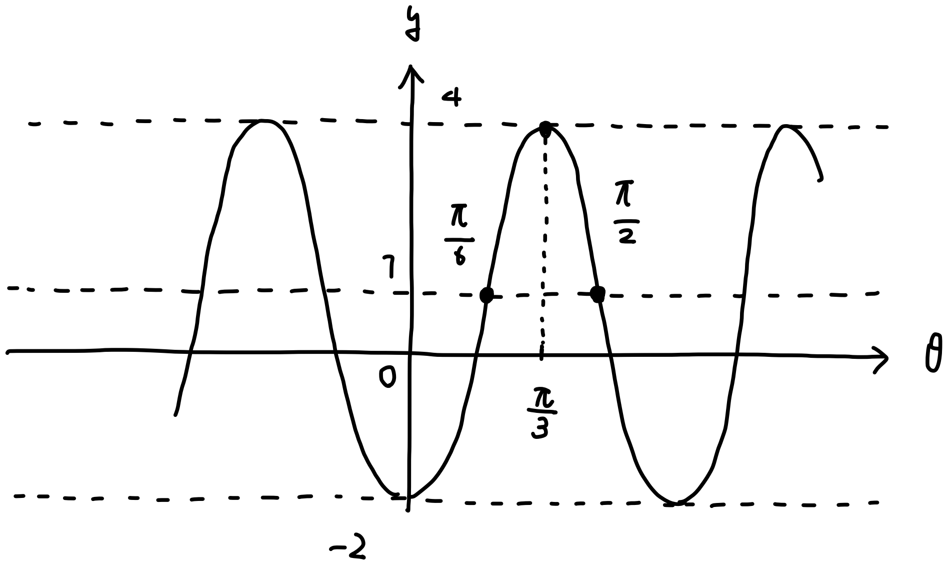
周期：π，$y=\sin\theta$ を θ 軸方向に $\frac{1}{2}$ 倍，

y 〃 2 倍

したグラフ。

3. $\alpha : \frac{3}{4}\pi$, $\beta : 3$, $\gamma : \frac{\pi}{2}$

4.



周期 : $\frac{2\pi}{3}$

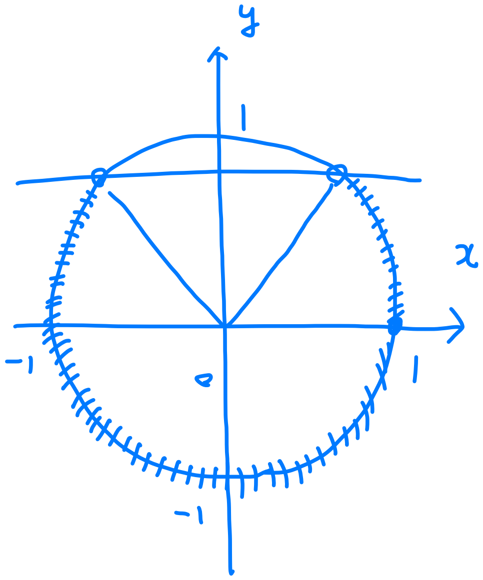
整数nに対し,

5. (1) $\theta = \frac{\pi}{3}, \frac{5}{3}\pi$, $n \in \mathbb{Z}$, $\theta = \frac{\pi}{3} + 2\pi \cdot n$, $\theta = \frac{5}{3}\pi + 2\pi \cdot n$

(2) $\theta = \frac{7}{6}\pi, \frac{11}{6}\pi$, $n \in \mathbb{Z}$, $\theta = \frac{7\pi}{6} + 2\pi \cdot n$, $\theta = \frac{11}{6}\pi + 2\pi \cdot n$

(3) $\theta = \frac{\pi}{6}\pi, \frac{7}{6}\pi$, $n \in \mathbb{Z}$, $\theta = \frac{\pi}{6} + \pi \cdot n$

6. (1) $0 \leq \theta < \frac{\pi}{3}, \ \frac{2}{3}\pi < \theta < 2\pi$ //



(2) $0 \leq \theta < \frac{3}{4}\pi, \ \frac{5}{4}\pi < \theta < 2\pi$ //

(3) $\frac{\pi}{2} < \theta < \frac{2}{3}\pi, \ \frac{3}{2}\pi < \theta < \frac{5}{3}\pi$ //

7. (1)

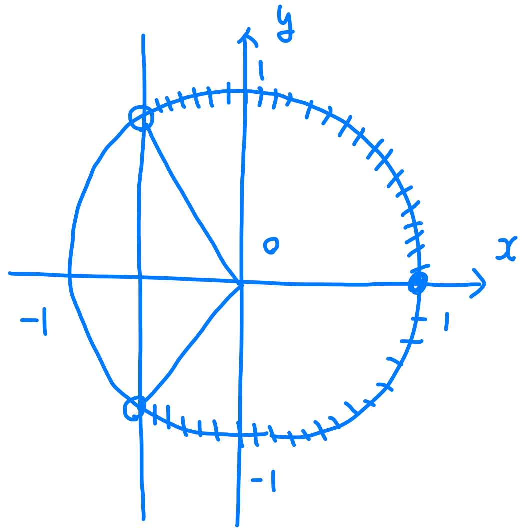
$$\sin 255^\circ = \sin(180^\circ - 255^\circ)$$

$$= \sin(-75^\circ)$$

$$= -\sin(45^\circ + 30^\circ)$$

$$= -\frac{1}{\sqrt{2}}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)$$

$$= -\frac{1+\sqrt{3}}{2\sqrt{2}}$$ //



(2)

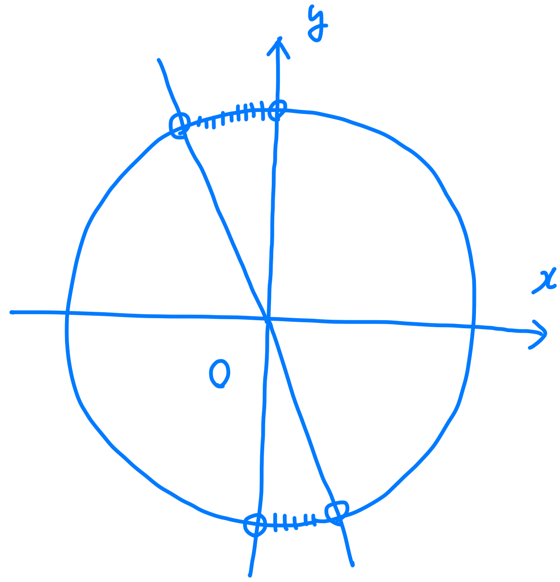
$$\tan 195^\circ = -\tan(180^\circ - 195^\circ)$$

$$= -\tan(-15^\circ)$$

$$= \tan(45^\circ - 30^\circ)$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \cdot \frac{1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$ //



7. (3)

$$\cos\frac{13}{12}\pi = \cos\left(\pi + \frac{\pi}{12}\right)$$

$$= -\cos\frac{\pi}{12}$$

$$= -\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$= -\left(\frac{1}{2}\cdot\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}\right)$$

$$= -\frac{\sqrt{3}+1}{2\sqrt{2}} \quad //$$

8. (1) $\cos\alpha = \frac{1}{2}$, $\sin\beta = \frac{4}{5}$

なので、

$$\sin(\alpha+\beta) = \frac{\sqrt{3}}{2}\left(-\frac{3}{5}\right) + \frac{1}{2}\cdot\frac{4}{5} = \frac{4-3\sqrt{3}}{10} \quad //$$

$$\cos(\alpha+\beta) = \frac{1}{2}\left(-\frac{3}{5}\right) - \frac{\sqrt{3}}{2}\cdot\frac{4}{5} = \frac{-3-4\sqrt{3}}{10}$$

$$\tan(\alpha+\beta) = \frac{4-3\sqrt{3}}{-3-4\sqrt{3}} \quad //$$

(2) $\sin\alpha = \cos\alpha = \frac{1}{\sqrt{2}}$, $\sin\beta = \frac{2}{\sqrt{5}}$, $\cos\beta = \frac{-1}{\sqrt{5}}$ $\tan\beta = \frac{2}{-1} = \frac{\frac{2}{\sqrt{5}}}{\frac{-1}{\sqrt{5}}}$

なので、

$$\tan(\alpha+\beta) = \frac{1+(-2)}{1-(-2)} = \frac{-1}{3} \quad //$$

$$\cos(\alpha-\beta) = \frac{1}{\sqrt{2}}\cdot\frac{-1}{\sqrt{5}} + \frac{1}{\sqrt{2}}\cdot\frac{2}{\sqrt{5}} = \frac{-1+2}{\sqrt{10}} = \frac{1}{\sqrt{10}} \quad //$$

9。 $\tan\alpha = \frac{3}{2}$, $\tan\beta = -5$ とすると、

$$\tan\theta = \frac{\frac{3}{2} - (-5)}{1 + \left(-\frac{3}{2}\cdot 5\right)} = \frac{3 + 10}{2 - 15} = -1$$

$\therefore\ \theta = \frac{\pi}{4}$ //

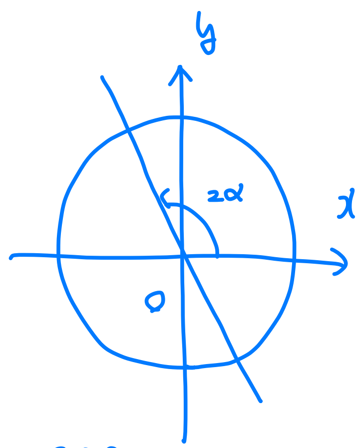
II ①。 (1) $\cos\alpha = -\frac{\sqrt{11}}{6}$ より、

$$\cos 2\alpha = \frac{1}{36}\left\{11 - 25\right\} = -\frac{7}{18}$$ //

$$\sin 2\alpha = -2\,\frac{5}{6}\,\frac{\sqrt{11}}{6} = -\frac{5\sqrt{11}}{18}$$ //

$$\tan 2\alpha = \frac{5\sqrt{11}}{7}$$ //

(2) $$\tan 2\alpha = \frac{8}{1 - 16} = \frac{-8}{15}$$ //



$$\frac{-8}{15} = \frac{+\frac{8}{\sqrt{225+64}}}{-\frac{15}{\sqrt{225+64}}}$$

$289 = 17^2$

$\therefore\ \cos 2\alpha = -\frac{15}{17}$, $\sin 2\alpha = \frac{8}{17}$ //

$\sin\alpha > 0$

$\wedge\ \cos\alpha > 0$

$\therefore\ \sin 2\alpha > 0$

10. (3) $\sin \frac{\pi}{12}$

$$= \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$= \frac{\sqrt{3}}{2} \frac{1}{\sqrt{2}} - \frac{1}{2} \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3} - 1}{2\sqrt{2}} \quad //$$

(4) $\sin \frac{\alpha}{2} = \sqrt{\frac{1 - \frac{2}{3}}{2}}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\therefore \sin^2\theta = \frac{1 - \cos 2\theta}{2}$

$$= \sqrt{\frac{3-2}{6}}$$

$$= \frac{1}{\sqrt{6}} \quad //$$

$$\cos \frac{\alpha}{2} = \frac{\sqrt{5}}{\sqrt{6}} \quad //$$

$$\tan \frac{\alpha}{2} = \frac{1}{\sqrt{5}} \quad //$$

(5) $\tan \alpha = \frac{5}{12} = \frac{-\frac{5}{13}}{-\frac{12}{13}}$

$12^2 + 5^2 = 144 + 25$

$= 169$

$= 13^2$

$\therefore \cos\alpha = -\frac{12}{13} \quad //$

$$\sin \frac{\alpha}{2} = \sqrt{\frac{1 + \frac{12}{13}}{2}} = \sqrt{\frac{13+12}{26}} = \frac{5}{\sqrt{26}} \quad //$$

$$\cos \frac{\alpha}{2} = -\frac{1}{\sqrt{26}} \quad // \qquad \tan \frac{\alpha}{2} = -5 \quad //$$

II1. (1) $\sin 2\alpha = 2\sin\alpha\cos\alpha$ なので,

$$(\sin\alpha - \cos\alpha)^2 = 1 - 2\sin\alpha \quad ▨$$

(2) $$\frac{2\tan\alpha}{\tan 2\alpha} = 1 - \tan^2\alpha$$

$$= \frac{\cos^2\alpha - \sin^2\alpha}{\cos^2\alpha}$$

$$= \frac{\cos 2\alpha}{\cos^2\alpha} \quad ▨$$

II2. (1) $-2\sin\theta + 2\cos\theta$

$$= 2\sqrt{2}\sin\left(\theta + \frac{3}{4}\pi\right) \quad //$$

(2) $2\sqrt{2}\sin\left(\theta - \frac{\pi}{6}\right)$

(3) $5\sin(\theta + \alpha)$,

ただし α は次を満たす:

$$\cos\alpha = \frac{3}{5}, \quad \sin\alpha = \frac{4}{5}. \quad //$$

Ⅱ3. $\sqrt{3}\sin\frac{\pi}{12}+\cos\frac{\pi}{12}$

$= 2\sin\left(\frac{\pi}{12}+\frac{\pi}{6}\right)$

$= \sqrt{2}$ //

Ⅱ4. $y=\sin x-\cos x \quad (0\leq x<2\pi)$

$= \sqrt{2}\sin\left(x-\frac{1}{4}\pi\right) \quad (0\leq x<2\pi)$

$\therefore\ x=\frac{3}{4}\pi$ のとき，最大値 $\sqrt{2}$，

$x=\frac{7}{4}\pi$ 〃 最小値 $-\sqrt{2}$ //