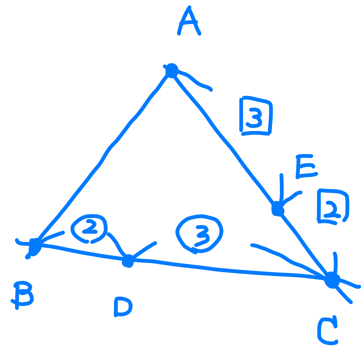
第8回答え

1. (1) $\dfrac{5}{8}\vec{a} + \dfrac{3}{8}\vec{b}$

(2) $\dfrac{5}{2}\vec{a} + \dfrac{-3}{2}\vec{b}$

2. (1)
$$\overrightarrow{DE} = \overrightarrow{AE} - \overrightarrow{AD}$$
$$= \frac{3}{5}\vec{c} - \left(\frac{3}{5}\vec{b} + \frac{2}{5}\vec{c}\right)$$
$$= -\frac{3}{5}\vec{b} + \frac{1}{5}\vec{c} \;//$$



(2) $\overrightarrow{AG} = \dfrac{1}{3}(\vec{b}+\vec{c})$ ← 点Aと原点は同じとみなす。

そのときの点Gの(原点に対する)

位置ベクトル。

3. A($\vec{a}$), B($\vec{b}$), C($\vec{c}$)

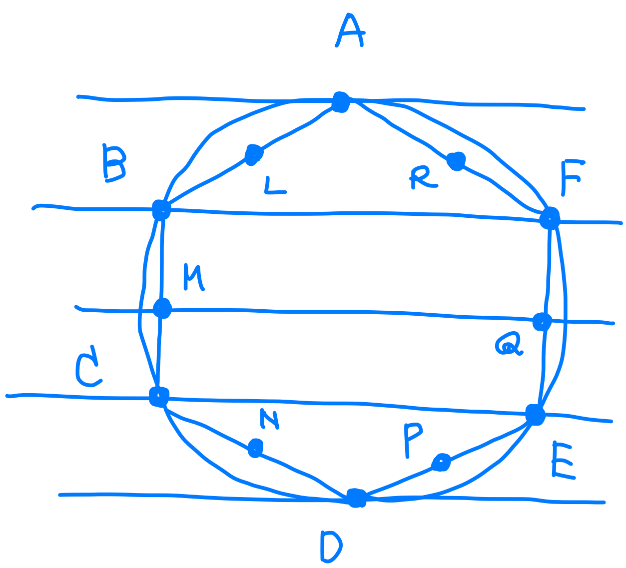
D($\vec{d}$), E($\vec{e}$), F($\vec{f}$)

に対し、△LNQの重心

の位置ベクトルは

$\dfrac{1}{6}(\vec{a}+\vec{b}+\vec{c}+\vec{d}+\vec{e}+\vec{f})$

これは△MPRの重心に等しい ■



補: $\dfrac{1}{6}(\vec{a}+\vec{b}+\vec{c}+\vec{d}+\vec{e}+\vec{f}) = \dfrac{1}{3}\left\{\overbrace{\dfrac{1}{2}(\vec{b}+\vec{c})}^{\overrightarrow{OM}} + \overbrace{\dfrac{1}{2}(\vec{d}+\vec{e})}^{\overrightarrow{OP}} + \overbrace{\dfrac{1}{2}(\vec{f}+\vec{a})}^{\overrightarrow{OR}}\right\}$

4. $\overrightarrow{AP} + \overrightarrow{BP} - 2\overrightarrow{CP} - 3\overrightarrow{GC}$

$= -\overrightarrow{PA} - \overrightarrow{PB} + 2\overrightarrow{PC} - 3(\overrightarrow{PC} - \overrightarrow{PG})$

$= -\overrightarrow{PA} - \overrightarrow{PB} - \overrightarrow{PC} + 3\overrightarrow{PG}$

$= \vec{0}$

$\therefore \overrightarrow{AP} + \overrightarrow{BP} - 2\overrightarrow{CP} = 3\overrightarrow{GC}$ ▨

5. $\begin{pmatrix} 1+1 \\ x-6 \end{pmatrix} = \begin{pmatrix} 2 \\ x-6 \end{pmatrix}$, $\begin{pmatrix} x+1 \\ 0-6 \end{pmatrix} = \begin{pmatrix} x+1 \\ -6 \end{pmatrix}$

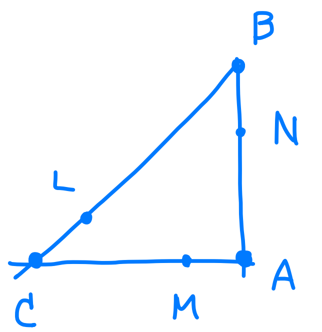
$2(-6) - (x-6)(x+1) = 0$ ← 3点が作る面積が0。

$x^2 - 5x - 6 + 12 = 0$

$(x-2)(x-3) = 0$

$\therefore x = 2, 3$ // ← 逆も成り立つ。

6. (ア) $\overrightarrow{AL} = \frac{2}{5}\vec{b} + \frac{3}{5}\vec{c}$



$\overrightarrow{NM} = \overrightarrow{AM} - \overrightarrow{AN}$

$= \frac{2}{5}\vec{c} - \frac{3}{5}\vec{b}$ //

6. (2) $\overrightarrow{AL}\cdot\overrightarrow{NM} = -\frac{6}{5^2}|\vec{a}|^2 + \frac{6}{5^2}|\vec{c}|^2$

$= 0 \quad (\because |\vec{a}|=|\vec{c}|,\ \vec{a}\cdot\vec{c}=0)$

$\therefore \overrightarrow{AL} \perp \overrightarrow{NM}$ ■

7. $s=0,\ t=\frac{3}{2}$ ← 実は一次独立の定ギと同じ。

8. 直線上の点P (x, y) に対し、

$$\begin{cases} x = 2 - t \\ y = -1 + 2t \end{cases} //$$

$$\begin{cases} x = 2 - t \\ y + 2x = 3 \end{cases}$$

$\therefore y = -2x + 3$ //

9. 直線上の点P (x, y) に対し、

$$\begin{cases} x = 2 + (2-1)t \\ y = 4 + (4+1)t \end{cases}$$

$$\begin{cases} x = 2 + t \\ y - 5x = -6 \end{cases}$$

$$\begin{cases} x = 2 + t \\ y = 4 + 5t \end{cases} //$$

$y = 5x - 6$ //

10. $(x-1)-2(y-2)=0$

11. (1) 円上の点P(x,y)に対し,

$$|\overrightarrow{CP}|^2 = |\overrightarrow{CA}|^2$$

$$(x-3)^2+(y-2)^2 = (3-1)^2+(2-1)^2$$

$$(x-3)^2+(y-2)^2 = 5 \quad //$$

(2) 円上の点P(x,y)に対し,

$$\left|\begin{pmatrix} x \\ y \end{pmatrix} - \frac{\begin{pmatrix} 1 \\ 4 \end{pmatrix}+\begin{pmatrix} 3 \\ 0 \end{pmatrix}}{2}\right|^2 = \left|\frac{1}{2}\begin{pmatrix} 1-3 \\ 4-0 \end{pmatrix}\right|^2$$

$$\frac{1}{2}\begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$\left|\begin{pmatrix} x-2 \\ y-2 \end{pmatrix}\right|^2 = 5$$

$$(x-2)^2+(y-2)^2 = 5 \quad //$$

12. $$\cos\alpha = \frac{\left|\begin{pmatrix} 4 \\ 2 \end{pmatrix}\cdot\begin{pmatrix} -3 \\ 1 \end{pmatrix}\right|}{\left|\begin{pmatrix} 4 \\ 2 \end{pmatrix}\right|\left|\begin{pmatrix} -3 \\ 1 \end{pmatrix}\right|}$$

$-12+2=-10$

$2\sqrt{4+1} \times \sqrt{10}$

$= 2\cdot 5\sqrt{2}$

$$= \frac{1}{\sqrt{2}}$$

$\therefore \alpha = 45^\circ \quad //$

II3. $L(-3,6,0)$, $M(0,6,-5)$, $N(-3,0,-5)$

II4. xy平面： $(-2,-3,-4)$

z軸： $(2,3,4)$

原点： $(2,3,-4)$

II5. $|\overrightarrow{AB}| = \sqrt{(1-3)^2+(-2-2)^2+(3+2)^2}$

$= \sqrt{4+16+25}$

$= 3\sqrt{5}$ //

II6. $\overrightarrow{AB} = \begin{pmatrix} 3-1 \\ 1-2 \\ 5-3 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$, $|\overrightarrow{AB}|^2 = 4+1+4 = 9$

$\overrightarrow{AC} = \begin{pmatrix} 2-1 \\ 4-2 \\ 3-3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$, $|\overrightarrow{AC}|^2 = 1+4 = 5$

$\overrightarrow{BC} = \begin{pmatrix} 1-2 \\ 2+1 \\ 0-2 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$, $|\overrightarrow{BC}|^2 = 1+9+4 = 14$

$\overrightarrow{AB} \cdot \overrightarrow{AC} = 0$

$\angle A = 90^\circ$ の直角三角形 //

17。 $P(0, y, z)$ とする。

$$|\overrightarrow{AP}|^2 = 9 + (y-1)^2 + (z-2)^2$$

$$|\overrightarrow{BP}|^2 = 1 + (y-3)^2 + z^2$$

$$|\overrightarrow{CP}|^2 = 4 + (y+1)^2 + (z-1)^2$$

$$\begin{cases} (y-1)^2 + (z-2)^2 + 9 = (y-3)^2 + z^2 + 1 \\ (y+1)^2 + (z-1)^2 + 4 = (y-3)^2 + z^2 + 1 \end{cases}$$

$$\begin{cases} -2y + 1 - 4z + 4 + 9 = -6y + 9 + 1 \\ 2y + 1 - 2z + 1 + 4 = -6y + 9 + 1 \end{cases}$$

$$\begin{cases} 4y - 4z + 4 = 0 \\ 8y - 2z - 4 = 0 \end{cases}$$

$$\begin{cases} y - z + 1 = 0 \\ 8y - 2z - 4 = 0 \end{cases}$$

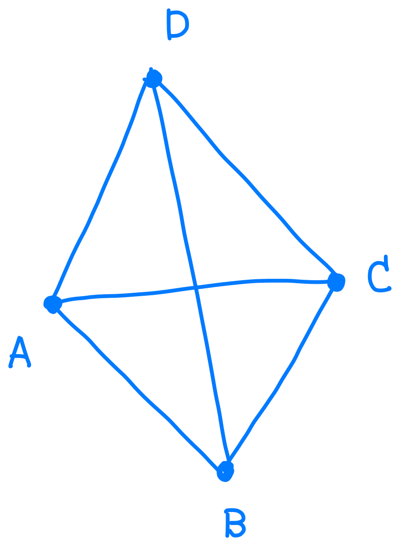
$$\begin{cases} y - z + 1 = 0 \\ 6y - 6 = 0 \end{cases}$$

$$\begin{cases} z = 2 \\ y = 1 \end{cases}$$

$\therefore\ P(0, 1, 2)$ //

Ⅱ8。 $\overrightarrow{AB} = \begin{pmatrix} 3-0 \\ 4-1 \\ -2+2 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix}$

$\overrightarrow{AC} = \begin{pmatrix} 0-0 \\ 4-1 \\ 1+2 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 3 \end{pmatrix}$



D (x, y, z) に対して,

$\overrightarrow{AD} = \begin{pmatrix} x \\ y-1 \\ z+2 \end{pmatrix}$

$\overrightarrow{AB} \cdot \overrightarrow{AD} = (3\sqrt{2})^2 \cdot \frac{1}{2}$

$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y-1 \\ z+2 \end{pmatrix} = 3$

$x + y - 1 - 3 = 0$

$x + y - 4 = 0$ 」

$\overrightarrow{AC} \cdot \overrightarrow{AD} = (3\sqrt{2})^2 \cdot \frac{1}{2}$

$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} x \\ y-1 \\ z+2 \end{pmatrix} = 3$

$y - 1 + z + 2 - 3 = 0$

$y + z - 2 = 0$ 」

$z + 2 = -y + 4$

Ⅱ8.

$$x^2+(y-1)^2+(z+2)^2=18$$

$$(y-4)^2+(y-1)^2+(y-4)^2=18$$

$33-18=15$

$$3y^2+(-8\ 2-2)y+16\cdot2+1-18=0$$

$$3y^2-18y+15=0$$

$$y^2-6y+5=0$$

$$(y-1)(y-5)=0$$

$$\therefore\ (3,1,1),\ (-1,5,-3)\ //$$

Ⅱ9. (1) $\overrightarrow{DG}=\overrightarrow{AG}-\overrightarrow{AD}$

$$=\vec{b}+\vec{e}+\vec{d}-\vec{d}$$

$$=\vec{b}+\vec{e}\ //$$

(2) $\overrightarrow{CE}=\overrightarrow{AE}-\overrightarrow{AC}$

$$=\vec{e}-\vec{d}-\vec{b}\ //$$

20. $\overrightarrow{AB} - \overrightarrow{CD} - \overrightarrow{AC} + \overrightarrow{BD}$

$$= \overrightarrow{AB} - \overrightarrow{AD} + \overrightarrow{AC} - \overrightarrow{AC} + \overrightarrow{AD} - \overrightarrow{AB}$$

$$= \vec{0}$$

$$\therefore \overrightarrow{AB} - \overrightarrow{CD} = \overrightarrow{AC} - \overrightarrow{BD} \quad \blacksquare$$

21.

$$2\vec{a} + 3\vec{b} = 2\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + 3\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2+0 \\ -2+6 \\ 4+3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} \text{ //}$$

22. $\overrightarrow{BC} = \begin{pmatrix} 2-1 \\ 1+1 \\ -1-1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ //

$$|\overrightarrow{BC}| = \sqrt{1+4+4} = \sqrt{9} = 3 \text{ //}$$

2B.

$$\begin{pmatrix}0\\3\\12\end{pmatrix} = s\begin{pmatrix}1\\2\\3\end{pmatrix} + t\begin{pmatrix}0\\2\\5\end{pmatrix} + u\begin{pmatrix}1\\3\\2\end{pmatrix}$$

$$\begin{cases}0 = s + u\\ 3 = 2s + 2t + 3u\\ 12 = 3s + 5t + 2u\end{cases}$$

$$\begin{cases}0 = s + u\\ 3 = 2t + u\\ 12 = s + 5t\end{cases}$$

$$\begin{cases}0 = s + u\\ 3 = 2t - s\\ 12 = 5t + s\end{cases}$$

$$\begin{cases}0 = s + u\\ 3 = 2t - s\\ 15 = 7t\end{cases}$$

$$\begin{cases}u = -s\\ s = \frac{30}{7} - \frac{21}{7}\\ t = \frac{15}{7}\end{cases}$$

$$\begin{cases}s = \frac{9}{7}\\ t = \frac{15}{7}\\ u = -\frac{9}{7}\end{cases}$$

$$\therefore \vec{p} = \frac{9}{7}\vec{a} + \frac{15}{7}\vec{b} - \frac{9}{7}\vec{c} \;//$$

$$\frac{9}{7}\begin{pmatrix}1\\2\\3\end{pmatrix} + \frac{15}{7}\begin{pmatrix}0\\2\\5\end{pmatrix} - \frac{9}{7}\begin{pmatrix}1\\3\\2\end{pmatrix}$$

$$= \frac{9}{7}\begin{pmatrix}0\\-1\\1\end{pmatrix} + \frac{15}{7}\begin{pmatrix}0\\2\\5\end{pmatrix}$$

$$= \frac{3}{7}\begin{pmatrix}0+0\\-3+10\\3+25\end{pmatrix}$$

$$= \frac{3}{7}\begin{pmatrix}0\\7\\28\end{pmatrix}$$

$$= 3\begin{pmatrix}0\\1\\4\end{pmatrix}$$