第9回答え

1。 (1) $(a+2)^3 = a^3 + 6a^2 + 12a + 8$

(2) $(2a-b)^3 = 8a^3 - 12a^2b + 6ab^2 - b^3$

(3) $(x+4)(x^2-4x+16) = x^3 + 64$ 〃

(4) $(5x-2y)(25x^2+10xy+4y^2) = 125x^3 - 8y^3$ 〃

(5) $(x+2y)^2(x^2-2xy+4y^2)^2 = (x^3+8y^3)^2$

$= x^6 + 16x^3y^3 + 64y^6$ 〃

2。 (1) $x^3+27 = (x+3)(x^2-3x+9)$

(2) $8a^3 - 27b^3 = (2a-3b)(4a^2+6ab+9b^2)$ 〃

(3) $64x^6 - y^6 = (4x^2-y^2)(16x^4+4x^2y^2+y^4)$

$= (2x+y)(2x-y)(16x^4+4x^2y^2+y^4)$

$= (2x+y)(2x-y)(4x^2+2xy+y^2)(4x^2-2xy+y^2)$, 3。 (1) $(x+1)^6$

$$= x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$$

(2) $(a-b)^6$

$$= a^6 - 6a^5b + 15a^4b^2 - 20a^3b^3 + 15a^2b^4 - 6ab^5 + b^6$$

(3) $\left(x + \frac{1}{3}\right)^5$

$$= x^5 + \frac{5}{3}x^4 + \frac{10}{3^2}x^3 + \frac{10}{3^3}x^2 + \frac{5}{3^4}x + \frac{1}{3^5} \quad 〃$$

4。 $3^3 (-2)^2 {}_5C_2$

$27 \cdot 4 \cdot 10$

$= 1080$ 〃

5。 $3^n = (1+2)^n$

$$= {}_nC_0 + 2\,{}_nC_1 + \cdots + 2^n\,{}_nC_n \quad \blacksquare$$

6. (1) $4x^3+4x^2+3x+2$

$$= (2x+1)(2x^2+x+1)+1$$

∴ 商 : $2x^2+x+1$

余り : 1 //

$$\begin{array}{r|rrrr} & & 2 & +1 & +1 \\ \hline 2\ +1 & 4 & +4 & +3 & +2 \\ & 4 & +2 & & \\ \hline & & +2 & +3 & \\ & & 2 & +1 & \\ \hline & & & +2 & +2 \\ & & & 2 & +1 \\ \hline & & & & 1 \end{array}$$

(2) $2x^3-3x-10$

$$= (2x^2+4x+5)(x-2)$$

∴ 商 : $x-2$

余り : 0 //

$$\begin{array}{r|rrrr} & & & 1 & -2 \\ \hline 2\ +4\ +5 & 2 & +0 & -3 & -10 \\ & 2 & +4 & +5 & \\ \hline & & -4 & -8 & -10 \\ & & -4 & -8 & -10 \end{array}$$

7. $Q = x^2+2x+9$

$R = 24$

$$x^3-x^2+3x-3 = (x-3)(x^2+2x+9)+24$$

$$\begin{array}{r|rrrr} & & 1 & +2 & +9 \\ \hline 1\ -3 & 1 & -1 & +3 & -3 \\ & 1 & -3 & & \\ \hline & & +2 & +3 & \\ & & 2 & -6 & \\ \hline & & & 9 & -3 \\ & & & 9 & -27 \\ \hline & & & & 24 \end{array}$$

8. (1) $A = (x^2-2x-1)(2x-3)-2x$

$= 2x^3-7x^2+4x-2x+3$

$= 2x^3-7x^2+2x+3$ //

(2) $6x^4+7x^3-9x^2-x+3 = B(2x^2+x-3)+6x$

$B(2x^2+x-3) = 6x^4+7x^3-9x^2-7x+3$

$\therefore B = 3x^2+2x-1$ //

$(2x^2+x-3)(3x^2+2x-1)$

$= 6x^4+(4+3)x^3+(2-2-9)x^2+\cdots$

9. (1) $\dfrac{12a^2b^4c}{16a^3bc^4}$

$= \dfrac{3b^3}{4ac^3}$ //

(2) $\dfrac{(x-2)(x-1)}{(x-3)(x-1)} = \dfrac{x-2}{x-3}$ //

(3) $\dfrac{a^2-(b+c)^2}{(a+b)^2-c^2} = \dfrac{\cancel{(a+b+c)}(a-b-c)}{(a+b-c)\cancel{(a+b+c)}} = \dfrac{a-b-c}{a+b-c}$ //

II ①. (1) $\dfrac{ax^2}{14a^3b^2} \times \dfrac{21a^2b}{3x}$

$= \dfrac{x}{2b}$ //

(2) $\dfrac{x^2-x-20}{x^3-2x+x} \times \dfrac{x^2-x}{x-5}$

$= \dfrac{(x+4)\cancel{(x-5)}\cancel{x}(x-1)}{\cancel{x}(x^2-2x+1)\cancel{(x-5)}}$

$= \dfrac{x+4}{x-1}$ //

(3) $\dfrac{a^2+a-6}{a^2-a} \times \dfrac{a^2+5a+6}{a^2+2a}$

$= \dfrac{(a+3)(a-2)\cancel{(a+2)}(a+3)}{a(a-1)\,a\cancel{(a+2)}}$

$= \dfrac{(a+3)^2(a-2)}{a^2(a-1)}$ //

II ①. (4) $\dfrac{x}{x+1} - \dfrac{1}{x+2}$

$= \dfrac{x(x+2) - (x+1)}{(x+1)(x+2)}$

$= \dfrac{x^2 + x - 1}{(x+1)(x+2)}$ //

(5) $\dfrac{x+8}{x^2+x-2} + \dfrac{x-4}{x^2-x}$

$= \dfrac{x+8}{(x+2)(x-1)} + \dfrac{x-4}{x(x-1)}$

$= \dfrac{(x+8)x + (x-4)(x+2)}{x(x-1)(x+2)}$

$= \dfrac{2x^2 + 6x - 8}{x(x-1)(x+2)}$

$= \dfrac{2(x+3x-4)}{x(x-1)(x+2)}$

$= \dfrac{2(x+4)}{x(x+2)}$ //

II ①. (6) $\dfrac{x+1}{1-1/(x+2)} + \dfrac{x+3}{1+1/(x+2)}$

$= \dfrac{\cancel{(x+1)}(x+2)}{\cancel{x+2}-1} + \dfrac{\cancel{(x+3)}(x+2)}{\cancel{x+2}+1}$

$= 2x+4$ //

II II. (2) と (4)

II ②. (1) $a=2$, $b=4$, $c=1$

(2) $a=1$, $b=4$, $c=4$

(3) $a=1$, $b=2$

(4) $\dfrac{a}{x+1} + \dfrac{b}{x-1} + \dfrac{c}{(x-1)^2}$

$= \dfrac{a(x-1)^2 + b(x+1)(x-1) + c(x+1)}{(x+1)(x-1)^2}$,

$(a+b)x^2 + (-2a+c)x + (a-b+c)$

$a+b=0 \wedge -2a+c=0 \wedge a-b+c=4$

$\therefore a=1$, $b=-1$, $c=2$ //

$2a+c=4$
$-2a+c=0$

$2c=4$

$c=2$

$-2a+2=0$

$a=+1$

$b=-1$

Ⅱ3. $(a+b)^2+(a-b)^2$

$$= 2a^2+2b^2$$

$$= 2(a^2+b^2) \quad \blacksquare$$

Ⅱ4. (1) $-(ab+bc+ca)$

$$= -ab-b(-a-b)-a(-a-b)$$

$$= +b^2+a^2+ab$$

$$\therefore\ a^2+ab+b^2 = -(ab+bc+ca) \quad \blacksquare$$

(2) $a^2(b+c)+b^2(c+a)+c^2(a+b)+3abc$

$$= -a^3-b^3+(a+b)^3+3ab(-a-b)$$

$$= 0 \quad \blacksquare$$

115. $\dfrac{ab+cd}{ab-cd}$

$$\frac{a}{b} = \frac{c}{d}$$

$$b = a\frac{d}{c}$$

$$d = c\frac{b}{a}$$

$$= \frac{a^2 \cancel{\frac{d}{c}} + c^2 \cancel{\frac{b}{a}}}{a^2 \cancel{\frac{d}{c}} - c^2 \cancel{\frac{b}{a}}}$$

$$= \frac{a^2 + c^2}{a^2 - c^2} \quad \blacksquare$$

116. $xy + 12 - 4x - 3y$

$$= (x-3)(y-4) > 0$$

$$(\because x-3 > 0 \wedge y-4 > 0)$$

$$\therefore xy + 12 > 4x + 3y \quad \blacksquare$$

117. (1) $a^2 + 11 - 6a$

$$= (a-3)^2 + 2 > 0$$

$$\therefore a^2 + 11 > 6a \quad \blacksquare$$

IV。(2) $4a^2 - 3b(4a - 3b)$

$= (2a - 3b)^2 \geq 0$

$\therefore\ 4a^2 \geq 3b(4a - 3b)$ ▨

等号成立条件は $2a - 3b = 0$

(3) $a^2 + 2ab + 2b^2$

$= (a + b)^2 + b^2 \geq 0$

$\therefore\ a^2 + 2ab + 2b \geq 0$ ▨

等号成立条件は $a = b = 0$

(4) $2(x^2 + 3y^2) - 5xy$

$= 2x^2 + 6y^2 - 5xy$

$= 2\left(x - \frac{5}{4}\right)^2 + \frac{23}{8}y^2 \geq 0$

$\therefore\ 2(x^2 + 3y^2) \geq 5xy$ ▨

等号成立条件は $x = y = 0$

Ⅱ8.

$$\left(\sqrt{9a+16b}\right)^2 - \left(3\sqrt{a}+4\sqrt{b}\right)^2$$

$$= 9a+16b-\left(9a+16b+24\sqrt{ab}\right)$$

$$= -24\sqrt{ab} < 0$$

$$\sqrt{9a+16b} > 0 \land 3\sqrt{a}+4\sqrt{b} > 0$$

$$\therefore \sqrt{9a+16b} < 3\sqrt{a}+16\sqrt{b} \quad \blacksquare$$

Ⅱ9. (1) $a+\dfrac{9}{a} \geq 2\sqrt{9}$

$$\therefore a+\frac{9}{a} \geq 6$$

等号成立条件は $a=3$

(2) $(a+2b)\left(\dfrac{2}{a}+\dfrac{1}{b}\right)$

$$= 2+2+4\cdot\frac{b}{a}+\frac{a}{b}$$

$$\geq 4+2\sqrt{4}$$

$$\geq 8$$

$\therefore (a+2b)\left(\dfrac{2}{a}+\dfrac{1}{b}\right) \geq 8$ 等号成立条件は

$a=2b$

20。 $|-1-5| = 6$

21。(1) $\frac{3}{8}(-1) + \frac{5}{8}\cdot 7$

$= \frac{32}{8}$

$= 4$

(2) $\frac{-1+7}{2} = 3$

(3) $\frac{3+35}{2} = 19$

(4) $\frac{-5-21}{2} = -13$

22。 (1) 5

(2) $(4+1)^2 + (15-3)^2$

$= 25 + 144$

$= 169$

$\therefore 13$

23。 $\overrightarrow{AB} = \begin{pmatrix} 4-5 \\ 0-2 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$

$\overrightarrow{AC} = \begin{pmatrix} 0-5 \\ 2-2 \end{pmatrix} = \begin{pmatrix} -5 \\ 0 \end{pmatrix}$

$\overrightarrow{BC} = \begin{pmatrix} -5+1 \\ 0+2 \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$

$|\overrightarrow{AB}|^2 = 5$

$|\overrightarrow{AC}|^2 = 25$

$|\overrightarrow{BC}|^2 = 16+4 = 20$

$\therefore \angle B = 90^\circ$ 答

24。 (1) 内分点 $\left(\frac{-1+10}{3}, \frac{4-4}{3} \right) = (3, 0)$ //

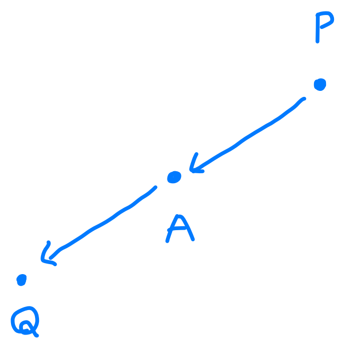
外分点 $(1+10, -4-4) = (11, -8)$ //

(2) $\left(\frac{-1+5}{2}, \frac{4-2}{2} \right) = (2, 1)$ //

25. $\frac{1}{3}\begin{pmatrix}0+1+2\\1-3-1\end{pmatrix} = \frac{1}{3}\begin{pmatrix}3\\-3\end{pmatrix}$

$\therefore (1,-1)$ //

26. $\overrightarrow{PA} = \begin{pmatrix}-1-3\\6-4\end{pmatrix} = \begin{pmatrix}-4\\2\end{pmatrix}$



$\begin{pmatrix}-1-4\\6+2\end{pmatrix}$

$\therefore Q(-5, 8)$ //