第11回 答え

以下では、

数列の初項をa, 等差数列の公差をd, 等比数列の公比をr

とする。

1。 (1) $10 = a + 18$

$a = -8$

$\therefore -8$ //

(2) $65 = 100 + 5d$

$13 - 20 = d$

$\therefore -7$ //

2。 (1) $\begin{cases} 10 + 5d = 20 \\ a + 4d = 10 \end{cases}$

$\begin{cases} d = 2 \\ a = 2 \end{cases}$

$\therefore 2n$ //

2. (2) $\begin{cases} 100 + 90d = 10 \\ a + 9d = 100 \end{cases}$

$\begin{cases} d = -1 \\ a = 109 \end{cases}$

$\therefore$ $110 - n$ //

3. $4 + 6k = 2k$

$k = -1$ //

4. (1) $5 \times (3 + 21) = 120$ //

(2) $13 \times (100 - 2 \times 25)$

$= 650$ //

5. $S_n = \frac{n}{2}\{4 + 2(n-1)\}$

$= n(n+1)$ //

$S_{10} = 110$ //

6. $\frac{n}{2}\left\{8-3(n-1)\right\}$

$= \frac{n}{2}(-3n+11)$ //

7. $\frac{7}{2}(85+43)$

$= 7\cdot 64$

$= 448$ //

$85-7(n-1)=43$

$-7n+92=43$

$7n=92-43$

$7n=49$

$n=7$

$85+43=128$

$7\times 64=420+28$

8. $\begin{cases} 20 = \sum_{k=1}^{5}\left\{a+(k-1)d\right\} \\ a+4d=12 \end{cases}$

$\begin{cases} 20 = 5a+\frac{5}{2}(0+4)d \\ a+4d=12 \end{cases}$

$4 = a+2d$

$\begin{cases} a+2d=4 \\ a+4d=12 \end{cases}$

$\begin{cases} a+2d=4 \\ 2d=8 \end{cases}$

$\begin{cases} a=-4 \\ d=4 \end{cases}$ //

← aとdについては

はじめに断った。

9. (1) $40+60+80+100$

$= 140 \times 2$

$= 280$ //

(2) $\sum_{k=30}^{100} k - \sum_{k=6}^{20} 5k$

$= \frac{100-30+1}{2} \times 130 - 5 \cdot \frac{20-6+1}{2} \times 26$

$= 13 \times \{71 \times 5 - 5 \cdot 15\}$

$= 65 \times \{71-15\}$

$= 3640$ //

$71-15 = 56$

$$\begin{array}{r} 56 \\ \times\ 65 \\ \hline 280 \\ 336 \\ \hline 3640 \end{array}$$

10. (1) 5×2^7

$= 10 \times 2^6$

$= 640$ //

(2) $160 = a \times (-2)^5$

$a = \frac{2^5 \times 5}{(-2)^5}$

$\therefore -5$ //

$160 = 16 \times 10$

$= 2^4 \times 2 \times 5$

II①. (3) $54 = 2r^3$

$r^3 = 27$

$\therefore 3$ //

III. (1) $\begin{cases} 972 = ar^5 \\ 36 = ar^2 \end{cases}$

$$\begin{array}{r} 27 \\ 36 \overline{)972} \\ 72 \\ \hline 252 \\ 252 \\ \hline 0 \end{array}$$

$\begin{cases} 27 = r^3 \\ 36 = ar^2 \end{cases}$

$\therefore \begin{cases} r = 3 \\ a = 4 \end{cases}$, $a_n = 4 \cdot 3^{n-1}$ //

(2) $\begin{cases} 12 = ar^2 \\ 192 = ar^6 \end{cases}$

$$\begin{array}{r} 16 \\ 12 \overline{)192} \\ 12 \\ \hline 72 \\ 72 \\ \hline 0 \end{array}$$

$\begin{cases} 12 = ar^2 \\ 16 = r^4 \end{cases}$

$\begin{cases} a = 3 \\ r = 2 \end{cases} \vee \begin{cases} a = 3 \\ r = -2 \end{cases}$ //

一般項は $a_n = 3 \cdot 2^{n-1}$ または $a_n = 3 \cdot (-2)^{n-1}$ //

Ⅱ2. (1) $4(k-1) = k^2$

$k^2 - 4k + 4 = 0$

$(k-2)^2 = 0$

$\therefore k = 2$ //

(2) $k(k+5) = 6^2$

$k^2 + 5k - 36 = 0$

$(k+9)(k-4) = 0$

$\therefore k = -9, 4$ //

Ⅱ3. $S_n = 4(2^n - 1)$, $S_{10} = 4 \times 1023 = 4092$

Ⅱ4. $64 \times 2 - 1$

$= 127$ //

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$= \frac{1}{1-r}\left\{a - ar^n\right\}$$

Ⅱ5. $\frac{-1}{1-(-2)}\left\{1-(-2)^n\right\}$

$= \frac{-1+(-2)^n}{3}$ //

16. (1) $2^5+2^6+2^7+2^8+2^9+2^{10}$

(2) $4^2+5^2+6^2+7^2$

17. $\sum_{k=1}^{8} k^2$

18. (1) $\sum_{k=1}^{n} 2\cdot 3^{k-1}$

$= 2\,\frac{3^n-1}{3-1}$

$= 3^n - 1$ //

(2) $\sum_{k=4}^{9} k^2 = \frac{1}{6}9\cdot10\cdot19 - \frac{1}{6}3\cdot4\cdot7$

$= \frac{3\cdot2}{6}\left\{3\,5\cdot19 - 14\right\}$

$= 271$ //

15×19

$=100+140+45$

$=285$

$285-14$

$=271$

(3) $\sum_{k=1}^{n}(k^2+2k)$

$= \frac{1}{6}n(n+1)(2n+1) + n(n+1)$

$= \frac{1}{6}n(n+1)\left\{2n+1+6\right\}$

$= \frac{1}{6}n(n+1)(2n+7)$ //

II 9。 (1) $\displaystyle\sum_{k=1}^{n}\{k\cdot(2k-1)\}$ $\underline{(k\text{項}:k(2k-1))}$ //

$$= \frac{2}{6}n(n+1)(2n+1) - \frac{1}{2}n(n+1)$$

$$= \frac{1}{6}n(n+1)\{2(2n+1)-3\}$$

$$= \frac{1}{6}n(n+1)(4n-1) \;//$$

(2) $\displaystyle\sum_{k=1}^{n}(2k-1)^3$ $\underline{(k\text{項}:(2k-1)^3)}$ //

~memo~

$$\forall k\in\mathbb{N},\ k^4-(k-1)^4 = k^4-(k^4-4k^3+6k^2-4k+1)$$

$$= 4k^3-6k^2+4k-1$$

$$\therefore\ n^4 = 4\sum_{k=1}^{n}k^3 - n(n+1)(2n+1) + 2n(n+1) - n$$

$$4\sum_{k=1}^{n}k^3 = n^4 + n(n+1)(2n+1) - 2n(n+1) + n$$

$$= n\{n^3+(n+1)(2n+1)-2(n+1)+1\}$$

ⅡB。 (2)

$$\sum_{k=1}^{n} k^3 = \frac{n}{4}\left\{n^3+2n^2+n\right\}$$

$$= \frac{n^2}{4}\left(n^2+2n+1\right)$$

$$= \frac{n^2}{4}\left(n+1\right)^2$$

$$= \left\{\frac{n}{2}(n+1)\right\}^2 \quad \lrcorner$$

$$\boxed{\therefore \sum_{k=1}^{n} k^3 = \sum_{k=1}^{n} k \cdot \sum_{k=1}^{n} k}$$

$$\sum_{k=1}^{n}(2k-1)^3 = \sum_{k=1}^{n}\left(8k^3-12k^2+6k-1\right)$$

$$= 2n^2(n+1)^2 - 2n(n+1)(2n+1) + 3n(n+1) - n$$

$$= n\left\{2n(n^2+\cancel{2n}+1) - 2(\cancel{2n^2}+3n+1) + 3n+3-1\right\}$$

$$= n\left\{2n^3-n\right\}$$

$$= n^2(2n^2-1) \quad //$$

20。 (1) $b_n = n^2$ //

与えられた数列を $\{a_n\}$ とすると,

$$a_n = \begin{cases} 1 & (n=1) \\ 1 + \sum_{k=1}^{n} k^2 & (n \geq 2) \end{cases}$$

$$= \begin{cases} 1 & (n=1) \\ 1 + \frac{1}{6} n(n+1)(2n+1) & (n \geq 2) \end{cases}$$

$$\therefore \quad \frac{1}{6}\left(2n^3 + 3n^2 + n + 6\right) \quad //$$

(2) $b_n = 3^{n-1}$ //

与えられた数列を $\{a_n\}$ とすると,

$$a_n = \begin{cases} 1 & (n=1) \\ 1 + \sum_{k=1}^{n-1} 3^{k-1} & (n \geq 2) \end{cases}$$

$$= \begin{cases} 1 & (n=1) \\ 1 + \frac{3^{n-1} - 1}{2} & (n \geq 2) \end{cases}$$

$$\therefore \quad \frac{3^{n-1} + 1}{2} \quad //$$

2Ⅱ。 求める数列の一般項をa_nとする。

(1)
$$\begin{cases} a_n = 1^3+2 & (n=1) \\ a_n = n^3+2-(n-1)^3-2 & (n \geq 2) \end{cases}$$

$$\begin{cases} a_n = 3 & (n=1) \\ a_n = 3n^2-3n+1 & (n \geq 2) \end{cases}$$ //

(2)
$$\begin{cases} a_n = 2^1+3 & (n=1) \\ a_n = 2^n+3-2^{n-1}-3 & (n \geq 2) \end{cases}$$

$$\begin{cases} a_n = 5 & (n=1) \\ a_n = 2^{n-1} & (n \geq 2) \end{cases}$$ //