第12回答え

1. (1) $a_n = 3 + 2(n-1)$

(2) $a_n = (-5)^{n-1}$

2. (1)
$$a_n = \begin{cases} 1 & (n=1) \\ 1 + \sum_{k=1}^{n-1} 4k & (n \geq 2) \end{cases}$$

$$= \begin{cases} 1 & (n=1) \\ 1 + 2(n-1)n & (n \geq 2) \end{cases}$$

$$= 2n^2 - 2n + 1 \quad //$$

(2)
$$a_n = \begin{cases} 1 & (n=1) \\ 1 + \sum_{k=1}^{n-1} 4^k & (n \geq 2) \end{cases}$$

$$= \begin{cases} 1 \\ 1 + \dfrac{4(4^{n-1}-1)}{4-1} & (n \geq 2) \end{cases}$$

$$= 1 + \frac{4^n - 4}{3} \quad //$$

B。 (1) $a_{n+1} = 3a_n - 2 \wedge a_1 = 2$

$a_{n+1} - 1 = 3(a_n - 1) \wedge a_1 = 2$

$a_n - 1 = 1 \cdot 3^{n-1}$

$\therefore a_n = 3^{n-1} + 1$ //

(2) $a_{n+1} = \frac{1}{3}a_n + 2 \wedge a_1 = 1$

$a_{n+1} - 3 = \frac{1}{3}(a_n - 3) \wedge a_1 = 1$

$\therefore a_n = -2\left(\frac{1}{3}\right)^{n-1} + 3$ //

4.

$$\forall n \in \mathbb{N},\quad \sum_{k=1}^{n}(3k-2) = \frac{1}{2}n(3n-1) \qquad (1)$$

を示す。

$$\forall l \in \mathbb{N},\quad \sum_{k=1}^{l}(3k-2) = \frac{1}{2}l(3l-1) \Rightarrow \sum_{k=1}^{l+1}(3k-2) = \frac{1}{2}(l+1)(3l+2) \qquad (2)$$

が示され:

$$\begin{aligned}
\sum_{k=1}^{l+1}(3k-2) &= \sum_{k=1}^{l}(3k-2) + 3l+1 \\
&= \frac{3}{2}l(l+1) + (l+1) \qquad \left(\because \sum_{k=1}^{l}(3k-2) = \frac{1}{2}l(3l-1)\right) \\
&= \frac{1}{2}(l+1)\left\{3l+2\right\},
\end{aligned}$$

(1)はn=1で成り立つので、

$$\begin{cases} \displaystyle\sum_{k=1}^{1}(3k-2) = 1 \\ \displaystyle\frac{1}{2}\cdot 1(3\cdot 1-1) = 1 \end{cases}, \qquad (3)$$

(2), (3)より (1)は示される ▨