第15回答え

1. 以下、積分定数をCとする。

Ⅱ. (1) $x^2 - 5x + C$

(2) $x^4 - x^3 + x + C$

(3) $\dfrac{t^3}{3} - \dfrac{t^2}{2} - 2t + C$

(4) $\dfrac{1}{4}(x+1)^4 + C$ ← 展開した形で、定数項がズレていてもよい。

Ⅲ. $f(x) = 2x^3 + x^2 + 3x + C$

であり、

$f(1) = 3$

なので、 $C = -3$

$\therefore f(x) = 2x^3 + x^2 + 3x - 3$ //

B_0 (1) $\left[x^3 + x^2 - x \right]_0^1$

$= 1$ //

(2) $\int_2^4 (x-1)(x-2)\,dx$

$= \int_2^4 (x-2+1)(x-2)\,dx$

$= \int_2^4 (x-2)^2\,dx + \int_2^4 (x-2)\,dx$

$= \frac{2^3}{3} + \frac{2^2}{2}$

$= \frac{14}{3}$ //

(3) $\left[\frac{t^3}{3} - \frac{3}{2}t^2 + 5t \right]_0^3$

$= 9 - \frac{27}{2} + 15$

$= \frac{1}{2}\left\{ 18 - 27 + 30 \right\}$

$= \frac{21}{2}$ //

B. (4) $\left[\frac{5}{3} y^3 + y^2 - y \right]_2^5$

$= \frac{5}{3}(125-8) + (25-4) - (5-2)$

$= \frac{1}{3}\left\{ 5 \cdot 117 + 3 \cdot 21 - 3 \cdot 3 \right\}$

$= \frac{1}{3}\left\{ 550 + 35 + 63 - 9 \right\}$

$98 - 9$

$= 89$

$550 + 89 = 639$

$= 213$ //

(5) $\int_3^0 (x^2 - 4x)\,dx$

$= -\int_0^3 x(x-4)\,dx$

$= -\frac{3^3}{3} + 2 \cdot 3^2$

$= 9$ //

(6) 0

(7) $\left[\frac{x^3}{3} + x \right]_0^3$

$= 12$ //

3. (8) $\int_{-1}^{0} (x^2-x)\,dx + \int_{-1}^{0} (2x-1)\,dx$

$= -\int_{0}^{-1} (x^2-x)\,dx - \left[x^2 - x \right]_{0}^{-1}$

$= -\frac{-1}{3} + \frac{1}{2} - (1+1)$

$= \frac{5}{6} - 2$

$= -\frac{7}{6}$ //

4. $3x^2 - 4x + 1$ //

5. (7) $\frac{d}{dx} \int_{a}^{x} f(t)\,dt = 2x+2$

$\therefore f(x) = 2x+2$ //

$\int_{a}^{x} (2t+2)\,dt$

$= \left[t^2 + 2t \right]_{a}^{x}$

$= x^2 + 2x + a^2 + 2a$

$\therefore a = 1, -3$ //

5. (2) $\frac{d}{dx}\int_1^x f(t)\,dt = 4x+1$

$\therefore\ f(x) = 4x+1$ //

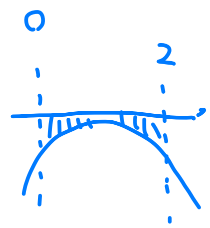
$\int_1^x (4t+1)\,dt = 2x^2 + x + a$

$\left[2t^2 + t\right]_1^x = 2x^2 + x + a$

$\therefore\ a = -3$ //

$x^2 - 2x + 2 = 0$

$(x-1)^2 = 0$



6. (1) $\int_0^2 (x^2 - 2x + 2)\,dx$

$= \left[\frac{x^3}{3} - x^2 + 2x\right]_0^2$

$= \frac{8}{3} - 4 + 4$

$= \frac{8}{3}$ //

(2) $\int_1^3 (x^3 + x)\,dx$

$= \left[\frac{x^4}{4} + \frac{x^2}{2}\right]_1^3$

$= \frac{1}{4}(81-1) + \frac{1}{2}(9-1)$

$= 20 + 4 = 24$ //

7. $\frac{1}{6}\left(2+\sqrt{3}-2+\sqrt{3}\right)^3$

x^2-4x+1

$= \frac{2^3}{6}\cdot 3\sqrt{3}$

$x = 2\pm\sqrt{3}$

$= 4\sqrt{3}$ //

8. (1) $S = -\int_2^3 (x-2)(x-3)\,dx$

$x^2-3x+5 = 2x-1$

$= \frac{1}{6}$ //

$x^2-5x+6 = 0$

$(x-2)(x-3) = 0$

(2) $S = -2\int_0^2 x(x-2)\,dx$

$x^2-3x-2 = -x^2+x-2$

$= 2\cdot\frac{1}{6}\cdot 2^3$

$= 2x^2-4x$

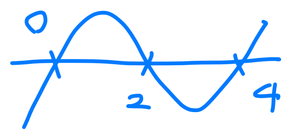
$= \frac{8}{3}$ //

$= 2x(x-2)$

9. $y = x^3 - 6x^2 + 8x$

$$= x(x^2 - 6x + 8)$$

$$= x(x-4)(x-2)$$



$$\therefore S = \int_0^2 x(x-4)(x-2)\,dx - \int_2^4 x(x-4)(x-2)\,dx$$

$$= \left[\frac{x^4}{4} - 2x^3 + 4x^2\right]_0^2 - \int_2^4 (x-2+2)(x-2-2)(x-2)\,dx$$

$$= 4 - 16 + 16 - \int_2^4 \left\{(x-2)^3 - 4(x-2)\right\}dx$$

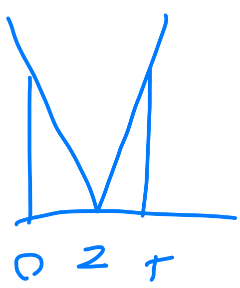
$$= 4 - \frac{2^4}{4} + 2 \cdot 2^2$$

$$= 8 \;/\!/$$

II①.

$$\int_0^5 |x-2|\,dx$$

$$= -\int_0^2 (x-2)\,dx + \int_2^5 (x-2)\,dx$$



$$= \int_2^0 (x-2)\,dx + \int_2^5 (x-2)\,dx$$

$$= \frac{4}{2} + \frac{3^2}{2}$$

$$= \frac{13}{2} \;/\!/$$