第22回 答え

II。 (1) $\cos\left(-\frac{5}{4}\pi\right) + i\sin\left(-\frac{5}{4}\pi\right)$

$= -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$ //

(2) $16\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)$

$= 16\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

$= 8\left(1 - \sqrt{3}i\right)$ //

(3) $\left\{\sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)\right\}^7$

$= 8\sqrt{2}\left(\cos\frac{7}{4}\pi + i\sin\frac{7}{4}\pi\right)$

$= 8\sqrt{2}\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)$

$= 8\left(1 - i\right)$ //

Ⅱ. (4) $\left(-\sqrt{3}+i\right)^{-6}$

$$= \left\{-2\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)\right\}^{-6}$$

$$= \frac{1}{64}\left(\cos\frac{-6}{6}\pi - i\sin\frac{-6}{6}\pi\right)$$

$$= -\frac{1}{64} \quad //$$

2. $\left(\cos\theta + i\sin\theta\right)^3$

$$= \cos^3\theta + 3\cos^2\theta \cdot i\sin\theta + 3\cos\theta \cdot \left(i\sin\theta\right)^2 + \left(i\sin\theta\right)^3$$

$$= \cos^3\theta - 3\sin^2\theta\cos\theta + i\left\{-\sin^3\theta + 3\cos^2\theta\sin\theta\right\}$$

$$= \cos^3\theta - 3\cos\theta + 3\cos^3\theta + i\left\{-\sin^3\theta + 3\sin\theta - 3\sin^3\theta\right\}$$

$$= 4\cos^3\theta - 3\cos\theta + i\left\{3\sin\theta - 4\sin^3\theta\right\}$$

$$\therefore \cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta \quad \blacksquare$$

B。 以下, $r, \theta \in \mathbb{R}$, $r > 0$, $\forall n \in \mathbb{Z}$ とする。

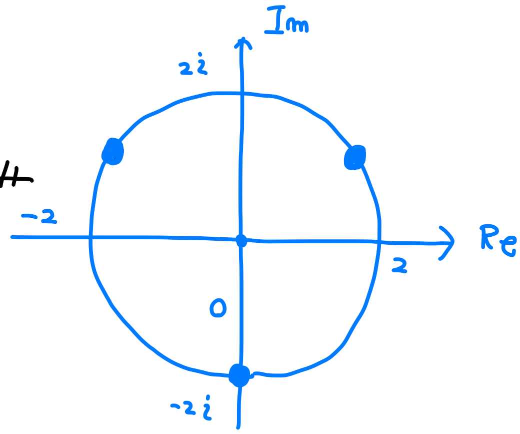
(1) $z = r(\cos\theta + i\sin\theta)$ とおくと,

$$z^3 = r^3(\cos 3\theta + i\sin 3\theta)$$

$$8i = 2^3\left\{\cos\left(\frac{\pi}{2} + 2\pi\cdot n\right) + i\sin\left(\frac{\pi}{2} + 2\pi\cdot n\right)\right\}$$

$$\therefore\ r = 2,\quad \theta = \frac{\pi}{6} + \frac{4\pi}{6}\cdot n \quad \leftarrow \frac{\pi}{6},\ \frac{5}{6}\pi,\ \frac{-3}{6}\pi$$

$$\therefore\ z = \sqrt{3} + i,\ -\sqrt{3} + i,\ -2i \quad //$$



(2) $z = r(\cos\theta + i\sin\theta)$ とおくと,

$$-1 - \sqrt{3}i = 2\left\{\cos\left(\frac{4}{3}\pi + 2\pi\cdot n\right) + i\sin\left(\frac{4}{3}\pi + 2\pi\cdot n\right)\right\}$$

より、 $r = \sqrt{2}$, $\theta = \frac{2}{3}\pi + \pi\cdot n$

$$\therefore\ z = \frac{\sqrt{2}}{2}\left(-1 + \sqrt{3}i\right),\ \frac{\sqrt{2}}{2}\left(1 - \sqrt{3}i\right) \quad //$$

4. (1) 3:2に内分する点：

$$\frac{2(4+2i)+3(-1+7i)}{5}$$

$$=\frac{1}{5}(5+25i)$$

$$=1+5i$$ //

2:3に内分する点：

$$\frac{3(4+2i)+2(-1+7i)}{5}$$

$$=\frac{1}{5}(10+20i)$$

$$=2+4i$$ //

(2) $$\frac{1}{2}(4+2i-1+7i)$$

$$=\frac{1}{2}(3+9i)$$ //

4. (3) 3:2に外分する点:

$$\frac{-2(4+2i)+3(-1+7i)}{3-2}$$

$$= -11+17i$$ //

2:3に外分する点:

$$\frac{3(4+2i)-2(-1+7i)}{-2+3}$$

$$= 14-8i$$ //

5. $\frac{1}{3}(2+i+5-i-4-4i)$

$$= \frac{1}{3}(3-4i)$$ //

6.

(1) 点 $-2+3i$ 中心, 半径 1 の円

(2) $|\bar{z}-i| = 2$

$\sqrt{(\bar{z}-i)(z+i)} = 2$

$|z+i| = 2$

∴ 点 $-i$ 中心, 半径 2 の円 //

(3) 2点 0, -4 を結ぶ線分の垂直二等分線

(4) 2点 $3-i$, -1 を結ぶ線分の垂直二等分線

6. (5) $|z+1| = 2|z-2|$

$\Leftrightarrow |z+1|^2 - 4|z-2|^2 = 0$

$\Leftrightarrow (z+1)(\bar{z}+1) - 4(z-2)(\bar{z}-2) = 0$

$\Leftrightarrow -3z\bar{z} + 9z + 9\bar{z} - 15 = 0$

$\Leftrightarrow z\bar{z} - 3z - 3\bar{z} + 5 = 0$

$\Leftrightarrow (z-3)(\bar{z}-3) - 4 = 0$

$\Leftrightarrow |z-3|^2 = 2^2$

$\Leftrightarrow |z-3| = 2$

$\therefore$ 点 3 中心，半径 2 の円 //

6。 (6) $3|z-i| = 2|z-1|$

$$\Leftrightarrow 9(z-i)(\bar{z}+i) - 4(z-1)(\bar{z}-1) = 0$$

$$\Leftrightarrow 5z\bar{z} + (4+9i)z + (4-9i)\bar{z} + 5 = 0$$

$97-25=72$

$$\Leftrightarrow 5\left(z + \frac{4-9i}{5}\right)\left(\bar{z} + \frac{4+9i}{5}\right) - \frac{16+81-25}{5} = 0$$

$$\Leftrightarrow \left|z + \frac{4-9i}{5}\right|^2 = \frac{36\cdot 2}{5^2}$$

∴ 点 $\frac{-4+9i}{5}$ 中心 半径 $\frac{6\sqrt{2}}{5}$ の円 //

7。 (7) 点i中心 半径1の円

P。 (2)

$$\begin{cases} |z| = 1 \\ w = \dfrac{iz+4}{2} \end{cases}$$

$$\Leftrightarrow \begin{cases} z\bar{z} = 1 \\ z = \dfrac{1}{i}(2w-4) \end{cases}$$

$$\Leftrightarrow \begin{cases} -2i(w-2)\cdot 2i(\bar{w}-2) - 1 = 0 \\ z = -2i(w-2) \end{cases}$$

$$\Leftrightarrow \begin{cases} 4|w-2|^2 = 1 \\ z = -2i(w-2) \end{cases}$$

$$\Leftrightarrow \begin{cases} |w-2| = \dfrac{1}{2} \\ z = -2i(w-2) \end{cases}$$

∴ 点2中心半径$\dfrac{1}{2}$の円 //

8.　$\beta - \alpha = 6 + \sqrt{3}i - (2 - \sqrt{3}i)$

$= 4 + 2\sqrt{3}i$,

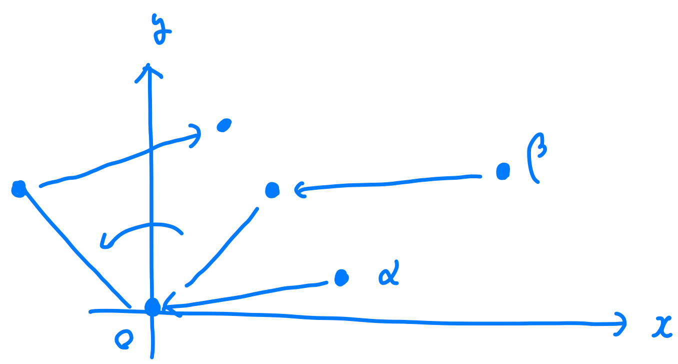
$2(2+\sqrt{3}i)\ \frac{1}{2}(1+\sqrt{3}i)$

$= 2 - 3 + i(\sqrt{3} + 2\sqrt{3})$

$= -1 + 3\sqrt{3}i$,

$\therefore\ -1 + 3\sqrt{3}i + 2 - \sqrt{3}i$

$= 1 + 2\sqrt{3}i$ //



Q.

$$\frac{6i-(\sqrt{3}+i)}{3\sqrt{3}+5i-(\sqrt{3}+i)} = \frac{-\sqrt{3}+5i}{2\sqrt{3}+4i}$$

$$= \frac{1}{2}\frac{(-\sqrt{3}+5i)(\sqrt{3}-2i)}{(\sqrt{3}+2i)(\sqrt{3}-2i)}$$

$$= \frac{1}{2}\frac{-3+10+7\sqrt{3}i}{-1}$$

$$= \frac{7}{2}(1-\sqrt{3}i)$$

$\angle BAC = 60^\circ$ //

面積は、 $\frac{1}{2}\frac{\sqrt{3}}{2}|2\sqrt{3}+4i||-\sqrt{3}+5i|$

$$= \frac{\sqrt{3}}{4}\cdot 2\cdot\sqrt{3+4}\cdot\sqrt{3+25}$$

$$= \frac{\sqrt{3}}{2}\cdot\sqrt{7}\cdot 2\sqrt{7}$$

$$= 7\sqrt{3}$$ //

Ⅱ⑩.

$$\frac{\alpha-\gamma}{\beta-\gamma}=\frac{5-i-(2+2i)}{3+i-(2+2i)}$$

$$=\frac{3-3i}{1-i}$$

$$=3$$

$$\therefore \frac{\alpha-\gamma}{\beta-\gamma}\in\mathbb{R}$$ ▨

ⅡⅡ.

$$\frac{\beta-\alpha}{\gamma-\alpha}=\frac{4+4i-(2+i)}{-1+3i-(2+i)}$$

$$=\frac{2+3i}{-3+2i}$$

$$=\frac{(2+3i)(-3-2i)}{9+4}$$

$$=\frac{1}{13}(-6+6-13i)$$

$$=-i$$

∴ 直線ABとACは直交 ▨