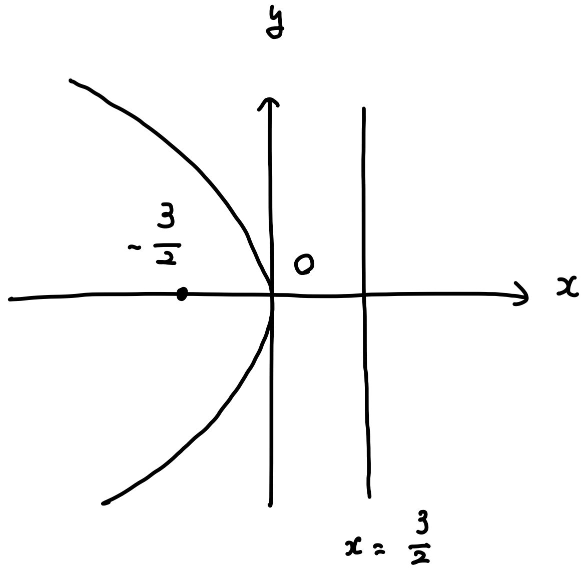
# 第23回 答え

Ⅱ。 (1) $y^2 = 4\left(-\frac{6}{4}\right)x$

焦点：$\left(-\frac{3}{2}, 0\right)$

準線：$x = \frac{3}{2}$

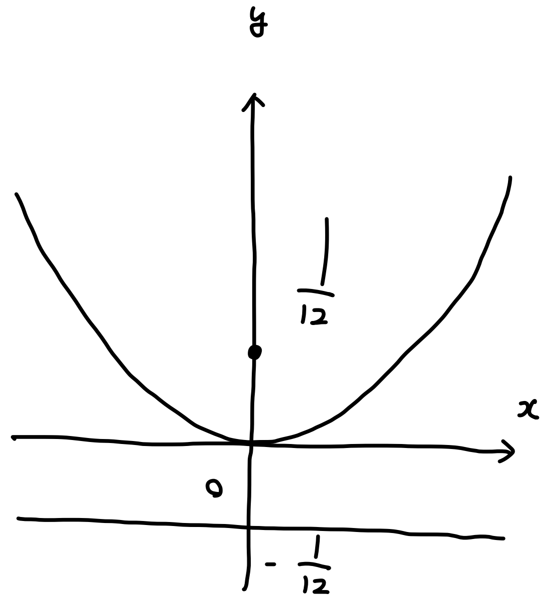



(2) $x^2 = \frac{1}{3}y$

$= 4 \cdot \frac{1}{12}y$

焦点：$\left(0, \frac{1}{12}\right)$

準線：$y = -\frac{1}{12}$




Ⅲ。 (1) $y^2 = 24x$

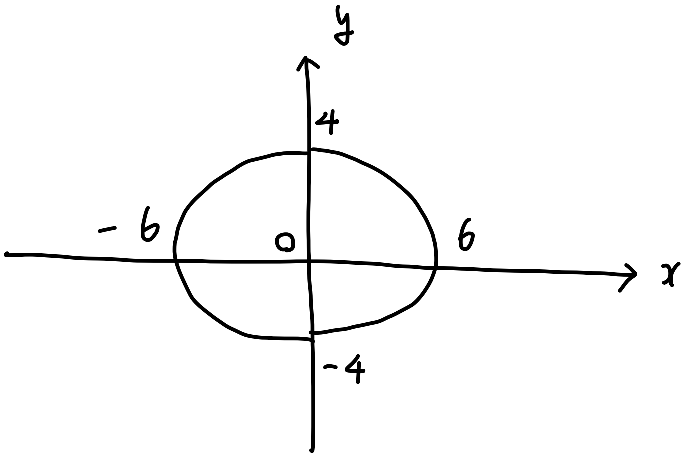
(2) $x^2 = -3y$

3.

(1) 長軸の長さ：12

短軸の長さ：8

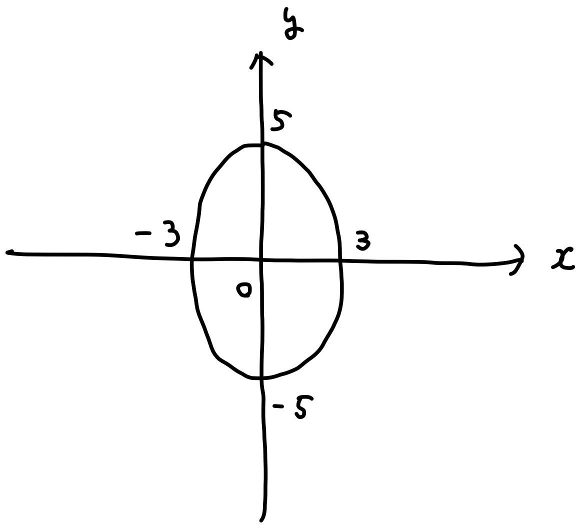
焦点：$(2\sqrt{5}, 0)$, $(-2\sqrt{5}, 0)$




(2) 長軸の長さ：10

短軸の長さ：6

焦点：$(0, 4)$, $(0, -4)$



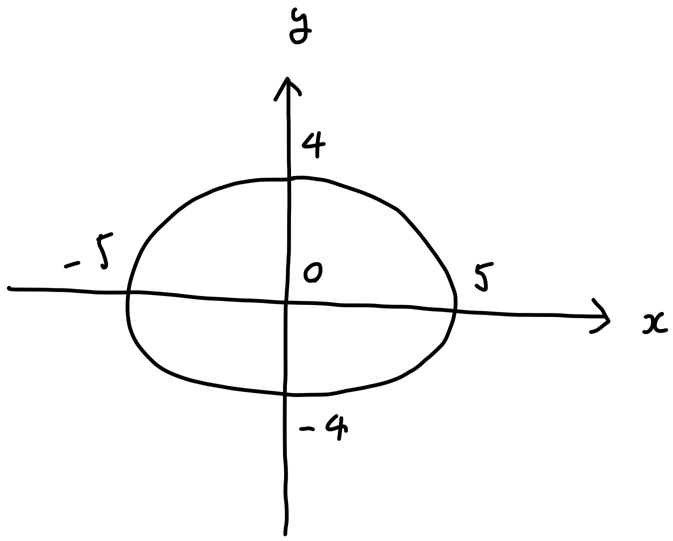
3。(3) $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$

$400 = 4 \times 100$
$= 16 \times 25$

長軸の長さ：10

短軸の長さ：8

焦点：$(3, 0)$, $(-3, 0)$




4。(ア) $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

4. (2) $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

とおき, 0でない実数 $a, b$ を求める:

$$\begin{cases} \dfrac{27}{4}a^{-2} + b^{-2} = 1 \\ \dfrac{9}{4}a^{-2} + 3b^{2} = 1 \\ a > b > 0 \end{cases}$$

$$\begin{cases} \dfrac{27}{4}a^{-2} + b^{-2} = 1 \\ 8b^{-2} = 2 \\ a > b > 0 \end{cases}$$

$$\begin{cases} 27a^{-2} + 1 = 4 \\ b = 2 \\ a > b > 0 \end{cases}$$

$$\begin{cases} a = 3 \\ b = 2 \\ a > b > 0 \end{cases}$$

.

$\therefore \dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$ //

4. (3) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

とおき，0でない実数 $a$, $b$ を求める：

$$\begin{cases} a > b > 0 \\ \sqrt{7} = \sqrt{a^2 - b^2} \\ b = 3 \end{cases}$$

$$\begin{cases} a > b > 0 \\ a = 4 \\ b = 3 \end{cases}.$$

$$\therefore \quad \frac{x^2}{16} + \frac{y^2}{9} = 1 \quad //$$

5. $x^2 + y^2 = 5^2$

$$\frac{x^2}{5^2} + \frac{y^2}{5^2} = 1 ,$$

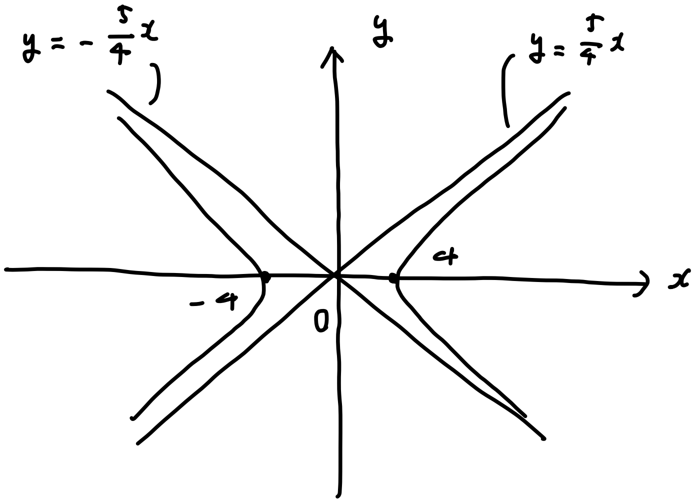
$y \to \frac{5}{4}y$ で

$$\frac{x^2}{5^2} + \frac{y}{4^2} = 1 \quad //$$

6. (1) 頂点：$(4, 0)$, $(-4, 0)$

焦点：$(\sqrt{41}, 0)$, $(-\sqrt{41}, 0)$

漸近線：$y = \frac{5}{4}x$, $y = -\frac{5}{4}x$




2つの実数 $a$, $c$ が $c > a > 0$ を満たし，

2点 $F(0, c)$, $F'(0, -c)$ からのキョリの差が $2a$

である点 $P(x, y)$ のキセキの eq. は，

$$\begin{cases} PF - PF' = \pm 2a \\ PF > 0 \land PF' > 0 \end{cases}$$

$$\begin{cases} PF = PF' \pm 2a \\ PF > 0 \land PF' > 0 \end{cases}$$

（続き）

$$\begin{cases} x^2 + y^2 - 2cy + c^2 = x^2 + y^2 + 2cy + c^2 + 4b^2 \pm 4b\,PF' \\ PF > 0 \land PF' > 0 \end{cases}$$

$$\begin{cases} -cy - b^2 = \pm b\,PF' \\ PF > 0 \land PF' > 0 \end{cases}$$

$$\begin{cases} c^2y^2 + b^4 + 2cb^2y = b^2\left(x^2 + y^2 + 2cy + c^2\right) \\ PF > 0 \land PF' > 0 \end{cases}$$

$$\begin{cases} b^2x^2 + (b^2 - c^2)\,y^2 = b^2(b^2 - c^2) \\ PF > 0 \land PF' > 0 \end{cases}$$

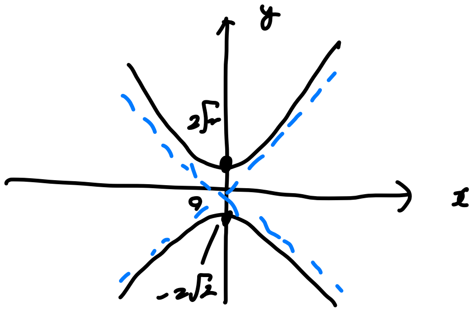
$a = \sqrt{c^2 - b^2}$ とおくと，

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1 \quad \text{（逆も成り立つ）}$$

6. (2) 頂点：$(0, 2\sqrt{2})$, $(0, -2\sqrt{2})$

焦点：$(0, 2\sqrt{3})$, $(0, -2\sqrt{3})$

漸近線：$y = \sqrt{2}x$, $y = -\sqrt{2}x$




$$\frac{x^2}{4} - \frac{y^2}{8} = -1$$

$$\frac{y^2}{8} = \frac{x^2}{4} + 1$$

$$y^2 = 8\left(\frac{x^2}{4} + 1\right)$$

$y > 0$ で

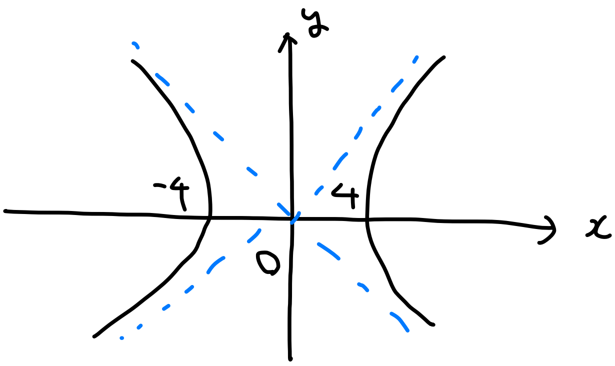
$$y = \frac{\sqrt{8}}{\sqrt{4}}\sqrt{x^2 + 4}$$

6. (3) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

頂点：$(4, 0), (-4, 0)$

焦点：$(5, 0), (-5, 0)$

漸近線：$y = \frac{3}{4}x,\ y = -\frac{3}{4}x$




7. (1) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

とおき，0でない実数 $a, b$ を求める：

$$\begin{cases} a > 0 \wedge b > 0 \\ 9 = a^2 + b^2 \\ \frac{16}{a^2} - \frac{25}{b^2} = -1 \end{cases}$$

$$\begin{cases} a > 0 \wedge b > 0 \\ a^2 + b^2 = 9 \\ \frac{16}{9 - b^2} - \frac{25}{b^2} = -1 \end{cases}$$

$-9 - (16 + 25)$

$= -25 \cdot 2$

$16b^2 - 25(9 - b^2) = -b^2(9 - b^2)$

$(b^2)^2 - 50b^2 + 9 \cdot 25 = 0$

$25 \pm \sqrt{25^2 - 9 \cdot 25}$

$16 \cdot 25$

$25 \pm 4 \cdot 5$

$45,\ 5$

7. (1) (続き)

$$\begin{cases} a>0 \wedge b>0 \\ a^2+45=9 \\ b^2=45 \end{cases} \vee \begin{cases} a>0 \wedge b>0 \\ a^2+5=9 \\ b^2=5 \end{cases}$$

$$\begin{cases} a>0 \wedge b>0 \\ a^2=4 \\ b^2=5 \end{cases}.$$

$$\therefore \frac{x^2}{4}-\frac{y^2}{5}=-1 \quad //$$

(2) $\frac{x^2}{3^2}-\frac{y^2}{b^2}=1$ を満たす実数$b$を求める.

$$\begin{cases} b>0 \\ 2=\frac{b}{3} \end{cases}.$$

$$\therefore \frac{x^2}{3^2}-\frac{y^2}{6^2}=1 \quad //$$

(3) $\frac{x^2}{4^2}-\frac{y^2}{4^2}=-1$

8. $\dfrac{(x-2)^2}{4} + \dfrac{(y+3)^2}{9} = 1$

焦点：$(2,\ \sqrt{5}-3)$, $(2,\ -\sqrt{5}-3)$ //

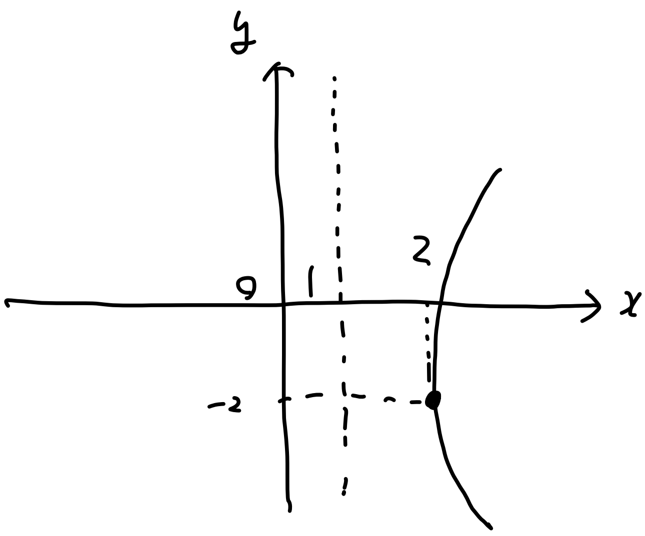
9. (ア) $y^2+4y-4x+12=0$

$(y+2)^2-4(x-2)=0$

∴ 放物線 $y^2=4x$ を $x$ 軸方向に $+2$,

$y$ 〃 $-2$

平行移動した曲線。




頂点：$(2,\ -2)$

焦点：$(3,\ -2)$

9. (2) $9x^2+4y^2-36x+8y+4=0$

$9(x^2-4x)+4(y^2+2y)+4=0$

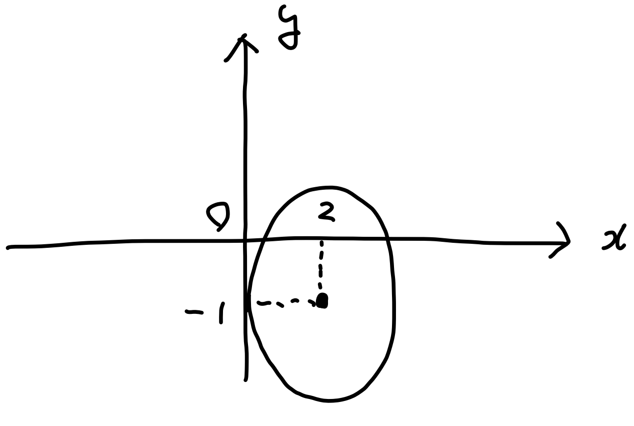
$9(x-2)^2+4(y+1)^2=36$

$$\frac{(x-2)^2}{4}+\frac{(y+1)^2}{9}=1$$

∴ 楕円 $\frac{x^2}{4}+\frac{y^2}{9}=1$ を $x$軸方向に $+2$

$y$ 〃 $-1$

だけ平行移動した曲線




中心：$(2,-1)$

焦点：$(2, \sqrt{5}-1)$, $(2, -\sqrt{5}-1)$