第24回 答え

Ⅱ。 (1)

$$\begin{cases} \dfrac{x^2}{9} + \dfrac{y^2}{4} = 1 \\ 2x - 3y = 0 \end{cases}$$

$$\begin{cases} 4x^2 + 9y^2 = 36 \\ 2x - 3y = 0 \end{cases}$$

$$\begin{cases} 2 \cdot 9y^2 = 36 \\ 2x - 3y = 0 \end{cases}$$

$$\begin{cases} y = \sqrt{2} \\ x = \dfrac{3\sqrt{2}}{2} \end{cases} \vee \begin{cases} y = -\sqrt{2} \\ x = -\dfrac{3\sqrt{2}}{2} \end{cases} \quad //$$

(2)

$$\begin{cases} y^2 = 6x \\ 2y - x = 6 \end{cases}$$

$$\begin{cases} y^2 = 12y - 36 \\ x = 2y - 6 \end{cases}$$

$$\begin{cases} y^2 - 12y + 36 = 0 \\ x = 2y - 6 \end{cases}$$

$$\begin{cases} (y-6)^2 = 0 \\ x = 2y - 6 \end{cases}$$

$$\begin{cases} y = 6 \\ x = 6 \end{cases} \quad //$$

2. (1) $\begin{cases} y = 2x + a \\ x^2 - y^2 = 1 \end{cases}$

$\begin{cases} y = 2x + a \\ x^2 - (2x+a)^2 = 1 \end{cases}$

$\begin{cases} y = 2x + a \\ -3x^2 - 4ax - a^2 = 1 \end{cases}$

$3x^2 + 4ax + a^2 + 1 = 0$ の判別式を D とすると

$\frac{D}{4} = 4a^2 - 3(a^2+1)$

$\therefore \quad a^2 - 3 > 0$

$\therefore \quad a < -\sqrt{3},\ \sqrt{3} < a$ //

2。 (2)

$$\begin{cases} y = mx + 3 \\ 4x^2 + 9y^2 = 36 \end{cases}$$

$$\begin{cases} y = mx + 3 \\ 4x^2 + 9(mx+3)^2 = 36 \end{cases}$$

$81 - 36$

$= 45$

$$\begin{cases} y = mx + 3 \\ (4+9m^2)x^2 + 54mx + 45 = 0 \end{cases}$$

求める条件は

$$(27m)^2 - 45\cdot(4+9m^2) = 0$$

$$9^{\cancel{2}}\overset{m^2}{\cancel{(9m^2 - 5m^2)}} = \cancel{9}\cdot 5\cdot \cancel{4}$$

$$\therefore m = \pm\frac{\sqrt{5}}{3}$$

$$\begin{cases} y = \pm\frac{\sqrt{5}}{3}x + 3 \\ x^2 \pm 2\sqrt{5}x + 5 = 0 \end{cases}$$

$\therefore$ $m = +\frac{\sqrt{5}}{3}$ のとき，接点は $\left(\sqrt{5}, \frac{4}{3}\right)$

$m = -\frac{\sqrt{5}}{3}$ 〃 $\left(-\sqrt{5}, \frac{4}{3}\right)$

2。(3) $\begin{cases} x + ky = 2 \\ y^2 = -8x \end{cases}$

$\begin{cases} x + ky = 2 \\ y^2 + 8(2 - ky) = 0 \end{cases}$

$y^2 - 8ky + 16 = 0$ の判別式を D とすると、

$\frac{D}{4} = (2k)^2 - 16 < 0$

$\therefore \quad k^2 < 4$

$-2 < k < 2$ //

3。 $\begin{cases} x + y = 1 \\ x^2 + 4y^2 = 4 \end{cases}$

$5y^2 - 2y - 3 = 0$

$(5y + 3)(y - 1) = 0$

$\begin{cases} x + y = 1 \\ (1-y)^2 + 4y^2 = 4 \end{cases}$

$\begin{cases} x + y = 1 \\ 5y^2 - 2y - 3 = 0 \end{cases}$

B。(続き)

$$\begin{cases} x+y=1 \\ y=-\frac{3}{5} \end{cases} \lor \begin{cases} x+y=1 \\ y=1 \end{cases}$$

$$\begin{cases} x=\frac{8}{5} \\ y=-\frac{3}{5} \end{cases} \lor \begin{cases} x=0 \\ y=1 \end{cases}$$

$$\vec{v}=\frac{1}{5}\begin{pmatrix} 8 \\ -3 \end{pmatrix}, \quad \vec{u}=\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\vec{v}-\vec{u}=\frac{1}{5}\begin{pmatrix} 8-0 \\ -3-5 \end{pmatrix}=\frac{1}{5}\begin{pmatrix} 8 \\ -8 \end{pmatrix}$$

$$|\vec{v}-\vec{u}|=\frac{8\sqrt{2}}{5},$$

$$\frac{\vec{v}}{2}+\frac{\vec{u}}{2}=\frac{1}{10}\begin{pmatrix} 8 \\ -3 \end{pmatrix}+\frac{1}{10}\begin{pmatrix} 0 \\ 5 \end{pmatrix}$$

$$=\frac{1}{10}\begin{pmatrix} 8+0 \\ -3+5 \end{pmatrix}$$

$$=\frac{1}{10}\begin{pmatrix} 8 \\ 2 \end{pmatrix}$$

$$=\frac{1}{5}\begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

$$\therefore \ \frac{8\sqrt{2}}{5}, \quad \left(\frac{4}{5}, \frac{1}{5}\right)$$ //

4。 楕円 $x^2+4y^2=4$ 上の点 (s,t) とすると,

$$\begin{cases} 0\cdot s+8\cdot t=4 \\ s^2+4t^2=4 \end{cases}$$

$$\begin{cases} t=1/2 \\ s=\sqrt{3} \end{cases} \vee \begin{cases} t=-1/2 \\ s=-\sqrt{3} \end{cases}$$

$$\therefore\ y=\frac{\frac{1}{2}-2}{\pm\sqrt{3}-0}x+2$$

$$=\frac{1-4}{\pm2\sqrt{3}}x+2$$

$$=\mp\frac{\sqrt{3}}{2}x+2 \quad //$$

5。 定数 k が

$$\begin{cases} y=2x+k \\ y^2=4x \end{cases}$$

$$\begin{cases} y=2x+k \\ y^2-2y+2k=0 \end{cases}$$

$y^2-2y+2k=0$ の判別式 D が 0 より

$1-2k=0$ $\quad\therefore\ y=2x+\frac{1}{2}$ //

6. (1)

$$\begin{cases} x = t-1 \\ y = t^2+2 \end{cases}$$

$$\begin{cases} x = t-1 \\ y = (t-1)^2+2(t-1)+3 \end{cases}$$

$$\begin{cases} x = t-1 \\ y = x^2+2x+3 \end{cases}$$

∴ 放物線 $y = x^2+2x+3$ //

(2)

$$\begin{cases} x = \sqrt{1-t^2} \\ y = t^2+1 \end{cases}$$

$$\begin{cases} 0 \leqq x \leqq 1 \\ x^2 = 1-t^2 \\ y = -(1-t^2)+2 \end{cases}$$

∴ 放物線 $y = -x^2+2 \ (0 \leqq x \leqq 1)$

7。 $y = -(x-t)^2 + t^2 + t^2 - 2t + 1$

$\therefore$ 放物線 $2x^2 - 2x + 1$ //

8。 (1) $\begin{cases} x = 3\cos\theta \\ y = 3\sin\theta \end{cases}$

(2) $\begin{cases} x = \dfrac{2}{\cos\theta} \\ y = \tan\theta \end{cases}$

(3) $\begin{cases} x = 4\cos\theta + 1 \\ y = 3\sin\theta - 2 \end{cases}$

$x = \dfrac{1}{\cos\theta}$

$y = \tan\theta$

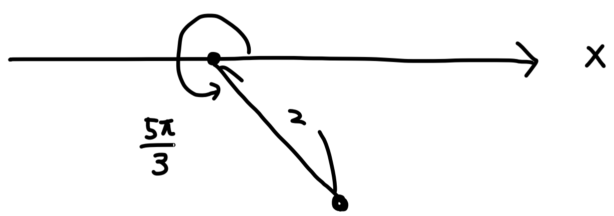
$x^2 - y^2 = \dfrac{1}{\cos^2\theta} - \tan^2\theta$

$= 1$

9。 (1) 楕円 $\dfrac{x^2}{9} + y^2 = 1$

(2) 双曲線 $(x+2)^2 - \dfrac{(y-3)^2}{4} = 1$

10。

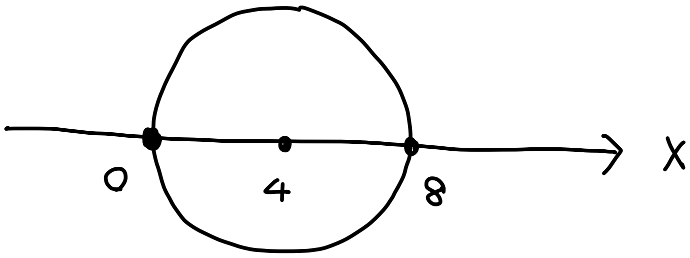


II1. $\left(4\cos\frac{-2\pi}{3},\ 4\sin\frac{-2}{3}\pi\right)$

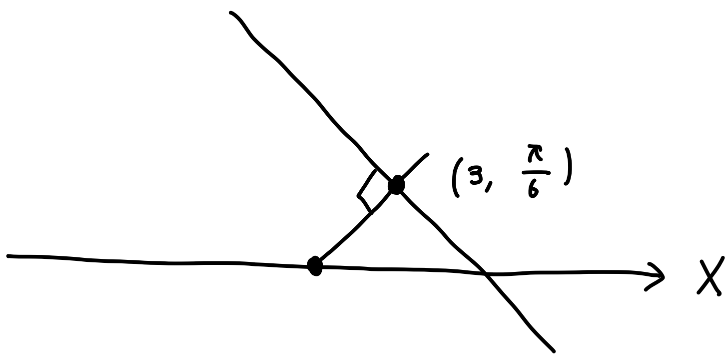
$= \left(-2,\ -2\sqrt{3}\right)$ 〃

II2. $\left(4,\ 4\pi/3\right)$ 〃

II3. (1)



(2)

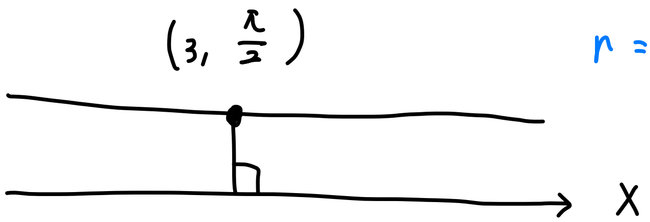


$\frac{\sqrt{3}}{2}$

〃

$r \cos\frac{\pi}{6} = 3$

(3)

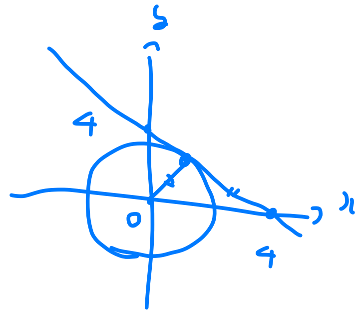


$r = 2\sqrt{3}$

Ⅱ4。 $r\cos(\theta - \frac{\pi}{3}) = a$

Ⅱ5。 $(x-1)^2 + y^2 = 1$

Ⅱ6。 (1) $r\cos(\theta - \frac{\pi}{4}) = \frac{4}{\sqrt{2}}$



(2) $(x^2 - 2)^2 + y^2 = 4$

$r = 4\cos\theta$ //

(3) $(r\sin\theta)^2 = -4\cos\theta$

$r^2\sin^2\theta + 4\cos\theta = 0$